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Analysis of Human Wrist Joint Impedance:

Does Human Joint Viscosity Depend on Its Angular Velocity?

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Abstract— **A novel model for the mechanical joint impedance, which takes into account the force-velocity relationships of skeletal muscles, is proposed. First, velocity-dependent viscosity changes are formulated using the equation proposed by A. V. Hill. Then, human wrist joint impedance is estimated by a conventional model (a linear model) and the proposed novel model (a nonlinear model) for comparison. The estimated moment of inertia indicates approximately constant value in all muscle contraction levels, and the joint stiffness estimated from the both models increases with the increment of muscle contractions. However, when joint angular velocity starts to change, the estimated joint viscosity dramatically decreases with the increment of joint angular velocity.**

Keywords—hill equation; joint impedance; skeletal muscle; velocity-dependent viscosity change; wrist

I. INTRODUCTION

Human voluntary movements in daily activities can be realized by contraction and slackness of skeletal muscles. These smooth movements become possible by contraction and slackness of the muscles with appropriate timing and cooperation. In other words, the control of the mechanical characteristics of the skeletal muscle is very important when a human carries out his/her voluntary movement smoothly.

The studies which analyze the mechanical characteristics of the skeletal muscle have been carried out in the physiology since the middle of the 1920s. Hill [1] examined shortening velocity-load characteristics on the sartorius muscle of the frog. He reported that there was a equilateral hyperbola relationship. Joyce et al. [2],[3] analyzed length-tension and shortening velocity-tension relationships of the skeletal muscle by changing rate of electrical stimuli applied to the cat soleus muscle. The results were as follows: (1) As the muscle length was longer, its tension was increased more nonlinearly. When the stimulus rate applied to the muscle was higher in the same length, its tension was increased. (2) As the shortening velocity of the muscle was increased, its tension was decreased nonlinearly. When the stimulus rate to the muscle became higher in the same shortening velocity, its tension was increased. These results suggest that the skeletal muscle has elasticity and viscosity. On the other hand, Mashima et al. [4] examined velocity-tension relationship of the frog semitendinosus muscle. They investigated the effects of the electrical stimulus applied to the skeletal muscle and the difference according to the direction of the contraction. They reported that the maximum shortening velocity of the skeletal muscle decreased with the increase of stimulus rate, and larger tension was observed by lengthening the skeletal muscle comparing with the one by shortening it. From the findings described above, it suggests that the skeletal muscle not only works as mere force generator but also has a nonlinear viscoelasticity.

While studies on dynamic properties of skeletal muscle were reported using animals, such dynamic properties have already been examined in humans. Wilkie [5] reported that the tension of the human flexor muscles in the upper arm depends on its shortening velocity. Moreover, the estimations of the mechanical impedance at a single joint have been carried out to clarify the control mechanism of the human joint impedance since the beginning of the 1980s. In the studies on a single joint impedance [6]-[10], all of them were applied disturbance around a joint, where it had small displacement and short duration. Therefore, it was assumed that the joint impedance while applying the external disturbance was indicated constant and no time dependency. Later, the changes of the joint stiffness during the elbow joint movements with the isotonic contraction were revealed in the experiments [11].

However, the angular velocity of the joint must change dramatically, even if the displacement applied to the joint appears very small. It has already indicated by Hill [1] that the shortening velocity-load relationship of the skeletal muscle becomes a equilateral hyperbola. So, it is clear that the viscosity corresponding to the derivative of this curve should change with the velocity. In other words, this is inconsistent with the assumption that the impedance parameter related to the velocity is constant during applying an external disturbance. From this point, we should use the impedance model which viscosity depends on the velocity in the analysis of the human impedance characteristics. However, there has been no studies that analyzed human movements using the impedance model positively taking into account the velocity-dependent characteristics of the skeletal muscle.

In this paper, we are going to carry out the following two points: (1) the development of a novel joint impedance model which takes into account for velocity-dependent viscosity changes; (2) the analysis of the wrist joint movement using the proposed model. The purpose of this study is to reveal how the impedance characteristics change with the isometric voluntary muscle contractions.

II. Human Wrist Joint Impedance Model

A. Tension-Velocity Relationship of Skeletal Muscle

Fig. 1(a) shows the relationship between the shortening velocity and the load using the sartorius muscle of frogs examined by Hill [1]. He approximated this relationship by the equilateral hyperbola as follows:

$$F = \frac{bF_{v=0} - av}{b + v}. \tag{1}$$

where F is the tension; v the shortening velocity of the skeletal muscle; $F_{v=0}$ the maximum isometric tension; and a, b represent constants, respectively. This equation is called as *Hill's characteristic equation*. As shown in Fig. 1(a), it is obvious that the tension decreases with the increase of the shortening velocity when the skeletal muscle shortens.

Fig. 1(b) shows the result by Mashima et al. [4] who examined the relationships between the tension and the velocity in the direction of shortening and lengthening for the semitendinosus muscle of frogs. As well as the result demonstrated by Hill, it reveals from Fig. 1(b) that the tension gradually decreases with the increase of the shortening velocity when the skeletal muscle shortens. However, it is clear that a bigger tension is produced than that in the shortening when the skeletal muscle is stretched, and the tension becomes almost constant after stretching to a certain extent. The analogical results are also revealed by Edman et al. [12] and Cole et al. [13] for such phenomena.

In this paper, we apply the Hill's characteristic equation in order to include the model in which the viscosity depends on the velocity in the joint impedance model. Based on the results obtained by Mashima et al. [4], we approximate the mechanical characteristics of the stretched muscle using the power-series which can express the nonlinearity and the steady state condition.

B. Dynamics of Skeletal Muscle

We consider the musculoskeletal system with one degree of freedom as Fig. 2. When the skeletal muscle is modeled by the contractile element, the variable elastic and viscous elements, the tension of the skeletal muscle F can be expressed as

$$F = f_{ce} - f_k - f_b, \tag{2}$$

where f_{ce}, f_k, f_b are the contractile force which contractile elements generate, the elastic and viscous forces, respectively. The elastic force f_k is expressed by the product of the muscle stiffness and the difference between the muscle length and the equilibrium point.

$$f_k = K(\alpha)(x - x_e), \tag{3}$$

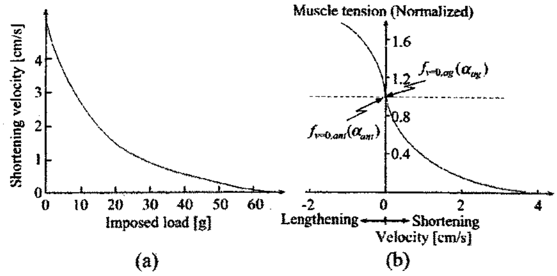


Figure 1. Tension-velocity relationships.

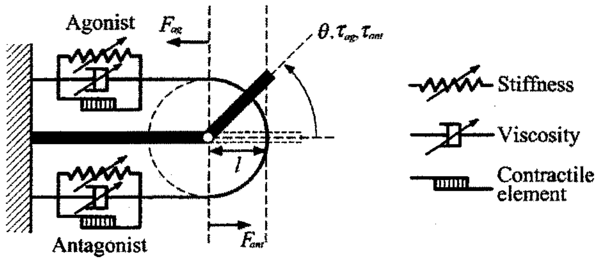


Figure 2. A single joint model.

where K, α, x, x_e represent the muscle stiffness, the muscle contraction level of the flexor ($0 \leq \alpha_f \leq 1$), the length of the muscle where the shortening direction is defined as the positive one, and the equilibrium point of the muscle, respectively. The muscle stiffness is assumed to change according to the muscle contraction level. The viscous force of shortening muscle f_b can be expressed as the subtraction from the muscle tension $f_{v=0}$ at $v = 0$ to that at the current velocity v, as shown in Fig. 1(b). Now, when $f_{v=0}$ is assumed to be proportional to the muscle contraction level, the viscous force can be expressed by using (1) as follows:

$$f_b = f_{v=0}(\alpha) - \frac{bf_{v=0}(\alpha) - av}{b + v}. \tag{4}$$

On the other hand, the viscous force of lengthening muscle f_b can be expressed as the subtraction from the muscle tension $f_{v=0}$ at $v = 0$ to that at the current extensor velocity v, in the same manner as the shortening muscle. In this paper, the changes of the viscous force with the lengthening muscle is expressed by using

$$f_b = c(\alpha)v^n, \tag{5}$$

where $c(\alpha)$ is a given real function which depends on the muscle contraction level of the extensor and n is a real constant.

The tension at the agonist's shortening F_{ag} can be expressed from (2)~(4) as follows:

$$\begin{aligned} F_{ag} = f_{ce,ag} &- K_{ag}(\alpha_{ag})(x_{ag} - x_{e,ag}) \\ &- f_{v_{ag}=0}(\alpha_{ag}) + \frac{bf_{v_{ag}=0}(\alpha_{ag}) - av_{ag}}{b + v_{ag}}. \end{aligned} \tag{6}$$

where the subscripts "ag" and "ant" represent the parameters of the agonist and the antagonist. When the agonists shorten, the antagonists lengthen. Therefore, the tension at the antagonist's shortening F_{ant} can be expressed from (2), (3), (5) as follows:

$$F_{ant} = f_{ce,ant} - K_{ant}(\alpha_{ant})(x_{ant} - x_{e,ant}) - c(\alpha_{ant}){v_{ant}}^n. \quad (7)$$

C. Joint Impedance Model

The joint impedance model is conducted from the impedance model of skeletal muscle formulated in section 2.2. When the length of moment arm of the agonists l is assumed to be the same as that of the antagonists, the torque generated by the muscle contraction can be expressed as follows:

$$\tau = lF. \quad (8)$$

where the direction of the agonists defined as positive, and that of antagonists is defined as negative. Therefore, the equation of motion around the single joint can be represented from (6)~(8) as follows:

$$\begin{aligned} I\ddot{\theta} &= \tau_{ag} + \tau_{ant} \\ &= \tau_{ce} - l^2\{K_{ag}(\alpha_{ag})(\theta - \theta_{e,ag}) \\ &+ K_{ant}(\alpha_{ant})(\theta - \theta_{e,ant})\} \\ &- \left\{\tau_{\dot{\theta}=0}(\alpha_{ag}) - \frac{b\tau_{\dot{\theta}=0}(\alpha_{ag}) - al^2\dot{\theta}}{b + l\dot{\theta}}\right\} \\ &- c(\alpha_{ant})l(l\dot{\theta})^n, \end{aligned} \quad (9)$$

where I is the moment of inertia around the wrist joint, τ_{ce} is the difference between the joint torque generated by the contractile element of the agonists $\tau_{ce,ag}$ and the joint torque generated by the contractile element of the antagonists $\tau_{ce,ant}$. α_{ag}, α_{ant} are constants and do not depend on time.

We consider that external torque τ_{ext} is applied to the wrist joint. When the onset time of τ_{ext} considers t_0 as shown in Fig. 4, we can obtain the following equation at a given time t:

$$\begin{aligned} \Delta\tau_{ext}(t) &= I\Delta\ddot{\theta}(t) + l^2(K_{ag}(\alpha_{ag}) + K_{ant}(\alpha_{ant}))\Delta\theta(t) \\ &+\tau_{\dot{\theta}=0}(\alpha_{ag}) - \frac{b\tau_{\dot{\theta}=0}(\alpha_{ag}) - al^2\Delta\dot{\theta}(t)}{b + l\Delta\dot{\theta}(t)} \\ &+c(\alpha_{ant})l(l\Delta\dot{\theta}(t))^n, \end{aligned} \quad (10)$$

where it is assumed that the equilibrium joint angles of the agonists and the antagonists, $\theta_{e,ag}$ and $\theta_{e,ant}$, and the joint torques generated by the contractile elements τ_{ce} are independent of time, that is, $\theta_{e,ag}(t_0) = \theta_{e,ag}(t)$, $\theta_{e,ant}(t_0) = \theta_{e,ant}(t)$, and $\tau_{ce}(t_0) = \tau_{ce}(t)$. Moreover, it is defined as $\Delta\tau_{ext}(t) = \tau_{ext}(t) - \tau_{ext}(t_0)$, $\Delta\theta(t) = \theta(t) - \theta(t_0)$, $\Delta\dot{\theta}(t) = \dot{\theta}(t) - \dot{\theta}(t_0)$, and $\Delta\ddot{\theta}(t) = \ddot{\theta}(t) - \ddot{\theta}(t_0)$. In the present study, we will estimate the joint impedance by using (10).

III. Experimental Method

A. Experimental Procedure

Fig. 3 illustrates experimental apparatus. Three male subjects (aged 23-35) participated in this experiment. Right hand of the subjects who sit in front of the table is fixed to the handle through the cast made from the glass fiber. In addition, subject's forearm is fixed to the arm supporting stand by the cuff.

The surface electromyogram (EMG) is measured from the agonist and antagonist muscles of wrist joint in the subject's forearm in order to clarify the action of the muscles around the wrist joint. A CRT display for on-line monitoring of the muscle contraction level is set in front of the subjects.

The task for the subjects utilizes the isometric flexion movement of wrist joint. During the experiment, subjects can regulate the muscle contraction level instructed by an experimenter, since the muscle contraction level is monitored by using the display. The Quick Release is performed by rapidly freeing the fixation of the handle after the muscle contraction reaches its steady-state level.

B. Experimental Apparatus

The Quick Release (QR) is a physiological technique adopted frequently to examine the relationships between velocity and tension in skeletal muscle [4].

When the load given to the skeletal muscle rapidly frees, the maximum shortening velocity is measured under the various load conditions. The force-velocity relationship of the skeletal muscle can be observed by plotting the measured data on various conditions. In the same way, the relationship between joint torque and joint angular velocity is revealed by rapidly freeing the external torque around the human single joint.

In this study, the linear motor table (Nihon Seiko Co., Ltd., encoder resolution: 1 [μm]) with one degree of freedom is used for the QR environment. At first, the handle position is controlled at the steady point just before the QR signal is applied. For the position control of the table, PID control given by (11) is used in order to compensate for steady-state deviation due to the friction of the table,

$$F_t = K_P(x^* - x) - B_P\dot{x} + K_I\int_0^t (x^* - x)dt, \quad (11)$$

where F_t is the driving force of the table, and K_P, B_P, K_I represent the feedback gain of position, velocity and integration, respectively. Also, x^*, x, $\dot{x}$ shows the desired trajectory, the handle position attached to the table and the handle velocity, respectively. Each gain used for the experiment is set at $K_P = 12000$ [N/m], $B_P = 250$ [Ns/m], $K_I = 30000$ [N/ms]. After the QR signal applied, the driving force of the table $F_{t,assist}$ is controlled using the following equation to compensate for the friction damping effect by the weight of the table:

$$F_{t,assist} = (M_e\hat{M} - 1)F_{subj} + \hat{B}\dot{x} + \hat{B}_c\frac{\dot{x}}{|\dot{x}|}, \quad (12)$$

where M_e and $\hat{M}$ are the equivalent inertia and the mass of the handle, F_{sub} is the force which the subjects applied to the force sensor, $\hat{B}$ is the viscous friction coefficient of the table, and $\hat{B}_c$ is the coulomb friction force of the table. From

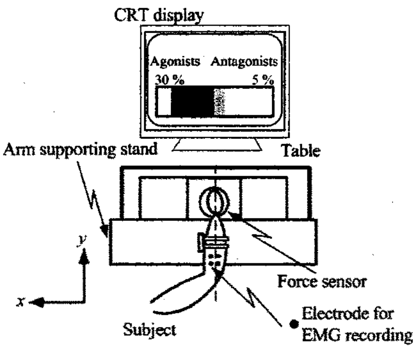


Figure 3. Experimental apparatus.

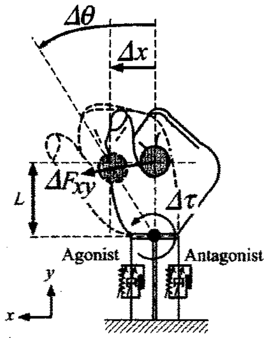


Figure 4. Discription of the wrist joint impedance for small displacement around an equilibrium posture.

the preliminary experiment, $\hat{M}$, $\hat{B}$ and $\hat{B}_c$ are estimated the following values: $\hat{M} = 7.5$ [kg], $\hat{B} = 29.5$ [Ns/m], $\hat{B}_c = 1.7$ [N]. The equivalent inertia of the handle is set as $M_e = 1.36$ [kg].

The force generated by the subject is measured by a six-axis force sensor (BL Autotec Co., Ltd., resolution ability: force x-axis, y-axis; 0.05 [N], z-axis; 0.15 [N])) attached at the handle of the table. The output from the encoder and force sensor are sampled at 2 [kHz].

From the slight hand displacement along the x axial direction $\Delta x(t)$ and the hand resultant force along the x and y axes $\Delta F_{xy}(t)$, we obtain the joint angle displacement $\Delta\theta(t)$ and joint torque displacement $\Delta\tau(t)$ by using the following equations:

$$\Delta\theta(t) = \sin^{-1}\frac{\Delta x(t)}{L}, \quad (13)$$

$$\Delta\tau(t) = L\Delta F_{xy}(t), \quad (14)$$

where L is the distance from the handle to the center of wrist joint (see Fig. 4).

C. Recording of Electromyography

The surface EMGs are recorded by the surface Ag/AgCl electrodes (The Japan GE Marquette Medical System Co.,Ltd., Biorode SDC-H). The electrodes are taped on the belly of the following muscles: Flexor Carpi Radialis (FCR), Flexor Carpi Ulnaris (FCU), Extensor Carpi Radialis (ECR) and Extensor Carpi Ulnaris (ECU). EMGs are amplified by

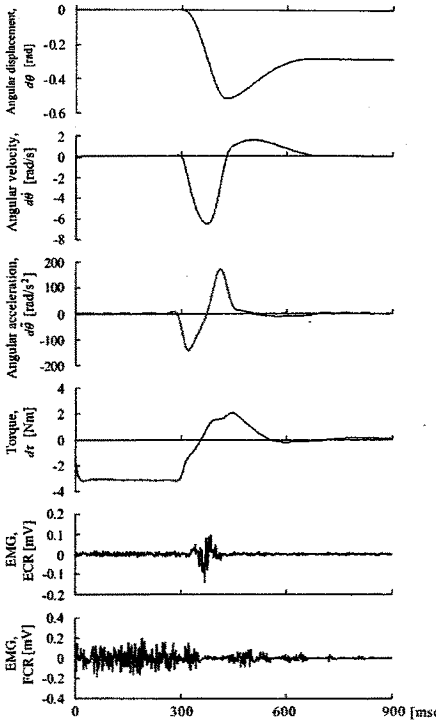


Figure 5. Specimen records in the QR experiment.

using the bioelectric amplifiers (Nihon Koden Co.,Ltd., AB-621G), and they are sampled into the computer at 2 [kHz].

Each muscle's contraction level is defined as follows: measured surface EMGs are rectified and integrated in the range of 200 [msec] before the QR, then normalized for the rectified and integrated value during the maximum voluntary contraction of each muscle. Flexors' (FCU and FCR) muscle contraction level is defined by summing up the rectified and integrated values of FCU and FCR, and normalized by the one during the maximum voluntary contraction. Extensors' (including ECU and ECR) muscle contraction level is normalized as well as the flexors' one.

D. Data processing

The parameter estimation of the nonlinear joint impedance model in (10) was carried out. Throughout the estimation, the parameters which minimized the root-mean-square errors between the measured value of the joint torque and the predicted value by the proposed model are searched. A quasi-Newton method is applied to the parameter estimation. The interactive process is terminated when the amount of change given by the root-mean-square errors became less than 1.0×10^{-11} [N^{2}m^2] for 5 consecutive iterations.

Fig. 5 represents the measured movements and the EMGs in the QR experiments. The interval reaching the maximum angular velocity from the minimum one is utilized for the impedance estimation (about 70∼120 [msec]). In Fig. 5, for example, 172 sampled data in the range from t=288 to 374 [msec] for the estimation were used.

TABLE I.

INITIAL VALUES FOR THE UNKNOWN PARAMETERS ESTIMATION.

$K_{ag}(\alpha_{ag}) + K_{ant}(\alpha_{ant})$ [N/m]	I [kgm^2]	a [N]
1.0×10^5	1.0×10^{-3}	1.0×10^2
b [m/s]	$c(\alpha)$	n
1.0×10^2	1.0×10^{-1}	1.0×10^{-1}

Table I summarizes the initial values of the unknown parameters. The initial values of the unknown parameters were given as follows: based on the results by Tsuji et al. [14] for the stiffness and the moment of inertia, by Chow and Darling [15] for a and b, and by Mashima et al. [4] for n. The particular constraints for each parameter are not added throughout this estimation in this study. In addition, the moment arm lengths of the agonists and antagonists are defined as 0.01[m] from the MRI image of a subject's forearm.

In this paper, target muscle contraction levels, α_{ag}, instructed to the subject are designed as the ones in every 5 % between 10 and 30 %MVC. The parameters are estimated for each 5 trials which produces the muscle contraction level closer to the corresponding target value, and the means and standard deviations for the estimated values are computed.

IV. RESULTS

A. Accuracy of Estimated Impedance

A preliminary experiment is carried out by using known physical objects to confirm whether the impedance is accurately measurable by using this experimental apparatus.

Fig. 6 shows the result of this experiment. The weight with the known mass is attached to the handle and the spring with the known elastic coefficient is installed between the handle and the environment. Then, their elastic coefficient (stiffness) and mass (inertia) are estimated simultaneously. In the figure, the intersections of the dotted lines show the true value, where the black dots show the estimated value by the experiment. From this figure, it is obvious that both of estimated stiffness and inertia are very close to their true values.

B. Estimated Wrist Joint Impedance

The wrist joint impedance parameters are estimated from the measured data of each subject in the QR experiments. The estimation of the wrist joint impedance parameters is performed by using a linear model and a nonlinear one. For the estimation using a linear model proposed by Tsuji et al. [14], data duration and the signal processing method are designed in the same way as the nonlinear model. Three subjects' data are analyzed by a linear least squares method.

Fig. 7 shows the results for one subject among three subjects. Fig. 7(a) shows the correlation coefficients between the predicted value by the estimated impedance parameters and the measured value. Although high correlation coefficients obtain from both models, the nonlinear model shows higher correlation coefficients than the linear model for all subjects. Figs. 7(b) and (c) show the result of the moment of inertia and the joint stiffness, respectively. The moments of inertia

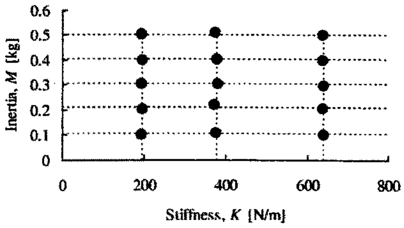


Figure 6. Accuracy of the impedance estimation using weights and springs. Means for 5 sets of the estimated results are plotted. Standard deviations for the estimated stiffness and inertia are less than 6.5 [N/m] and 0.007 [kg], respectively.

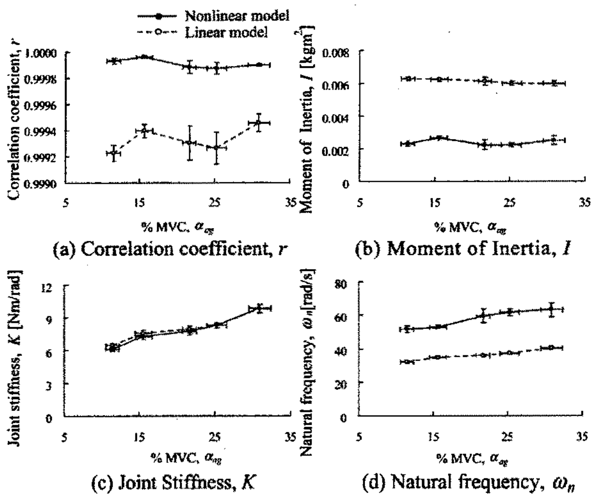


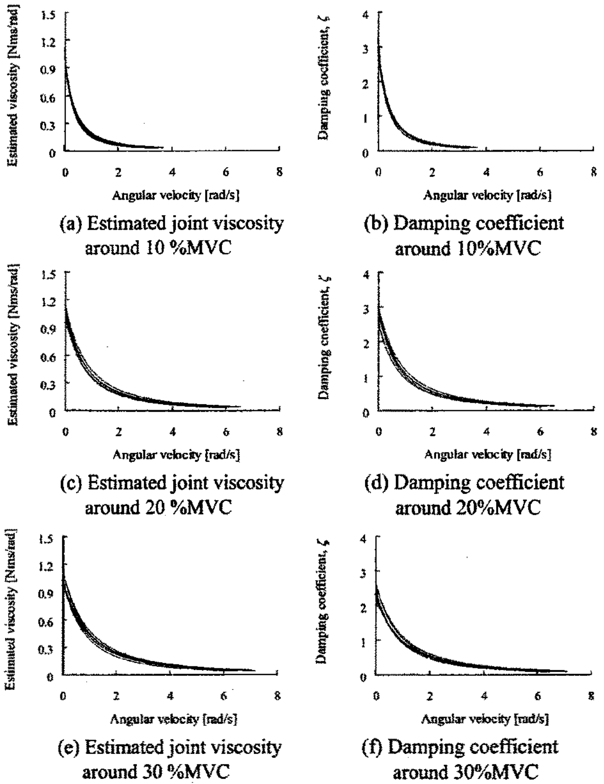
Figure 7. The correlation coefficient between estimated and measured torques, the moment of inertia, the joint stiffness, and the natural frequency for Subject B.

of both models do not depend on the muscle contraction levels. However, the means of moment of inertia estimated by the linear model is about 0.006 [kgm^2], and the means of moment of inertia estimated by the nonlinear model is about 0.003 [kgm^2].

On the other hand, as shown in Fig. 7(c), the joint stiffness by the linear model increases with the increment of the muscle contraction levels as well as the estimated results by the nonlinear model. The joint stiffness by the linear model almost agrees with that estimated by the nonlinear model. Moreover, the natural frequency of wrist joint ω_n is computed by the moment of inertia I and the joint stiffness K. The relationship between the natural frequency and the muscle contraction levels is shown in Fig. 7(d). The natural frequency of wrist joint also increases with the increment of muscle contractions, and the results obtained from the nonlinear model are approximately 20 [rad/s] higher than that from the linear model.

C. Changes of joint viscosity with joint angular velocity

Fig. 8 shows the estimated results of the joint viscosity and the damping coefficient. This figure shows the results around the 10, 20, and 30 %MVC levels for five trials. The joint viscosity is given by differentiating the third and forth terms in the right-hand side of (9) with respect to the joint



(a) Estimated joint viscosity around 10 %MVC
(b) Damping coefficient around 10%MVC
(c) Estimated joint viscosity around 20 %MVC
(d) Damping coefficient around 20%MVC
(e) Estimated joint viscosity around 30 %MVC
(f) Damping coefficient around 30%MVC

Figure 8. The estimated joint viscosity and damping coefficient corresponding to the 10, 20, 30 %MVC of agonist muscles for Subject B.

angular velocity $\dot{\theta}$. The joint viscosity of the agonists and the antagonists, B_{ag} and B_{ant} are expressed as follows:

$$B_{ag} = \frac{bl\tau_{\dot{\theta}=0}(\alpha_{ag}) + al}{(b + l\dot{\theta})^2}, \quad (15)$$

$$B_{ant} = n \cdot c(\alpha_{ant}) l^{n+1} \dot{\theta}^{n-1}. \quad (16)$$

The net of the joint viscosity is computed by the sum of these equations. From Fig. 8(a), (c), and (e) calculated by (15) and (16), the joint viscosity decreases with the increase of the joint angular velocity when the changes of the joint angular velocity start. This tendency is similar to that of the various muscle contraction levels. However, the damping coefficient at $\dot{\theta}$=0 have the tendency that they decrease with the increase of the muscle contraction levels.

These tendencies of the experimental results are similar in all of the subjects.

V. Conclusion

In this paper, the novel joint impedance model with a view to the force-velocity relationships of skeletal muscles has been proposed based on the nonlinear mechanical characteristics of the skeletal muscle. The distinguished findings of this paper can be summarized that the joint viscosity decreases when the joint velocity increases, and it becomes the minimum value at the maximum joint velocity. This tendency leads to the stability of the musculo-skeletal system during slow movements in order to achieve precise positioning easily. Conversely, the viscous coefficient of the musculo-skeletal system may decrease in order to prevent energy loss during fast movements. Although the experiments in this paper are performed according to the changes of the muscle contraction levels of the agonists, the future research will be directed to examine the changes of joint viscosity characteristics during the co-contraction of the antagonists and the dynamic movements.

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