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Rolling Based Manipulation for Multiple Objects

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Abstract

This paper discusses the manipulation of multiple objects under rolling contacts. For manipulating multiple objects, there are two key issues which do not arise in the manipulation of a single object, (1)each object's motion is restricted by the other objects, and (2)the contact force among objects is not controlled directly. As for (1), we first formulate the motion constraint for the whole grasp system, and then provide a necessary condition for manipulating multiple objects uniquely. As for (2), we provide a condition for determining the contact forces among objects uniquely. We further show a sufficient condition for manipulating multiple objects within the object motion constraint. Under this sufficient condition, we propose a control scheme for object motion by taking the motion constraint into account. An experimental result is provided to confirm our idea.

1 Introduction

Multifingered robot hands have potential advantages in performing various tasks with the dexterity of human hands. While much research has been done on multifingered robot hands, most of works have implicitly assumed that a multifingered hand manipulates only one object. Under such a condition, several grasping issues have been studied.

Let us now consider the case where a multi-fingered hand approaches and envelopes two cylindrical objects with significant friction[9] as shown in Fig.1. For two objects satisfying the rolling contact each other, we can expect that a multifingered hand can easily achieve an enveloping grasp by simply pushing two links contacting with the objects. During the lifting phase, links and two objects behave as if they were just connected by mechanical gears. Due to this mechanical properties, achieving an enveloping grasp for two objects seems to be even easier than for a single object under significant friction. This is a potential advantage for manipulating two objects, simultaneously.

We can find another advantage for manipulating multiple objects. Let us consider the case where a human picks up and transfers small objects such as coins, beans or such like from a table. In such a case, a human often grasps more than one object and manipulate them case by case. Generally, we can expect that treating multiple objects simultaneously makes it possible to achieve a handling task efficiently. These

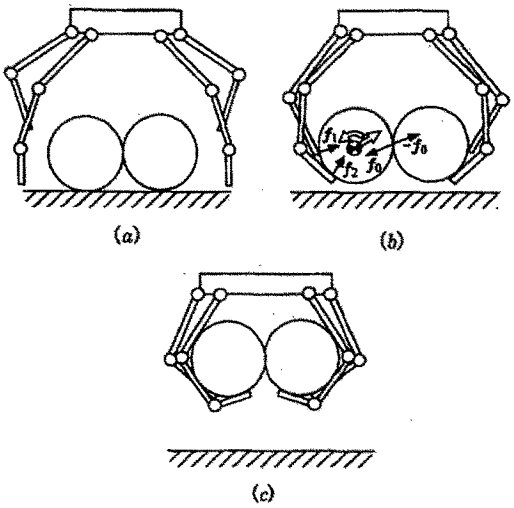

Fig. 1: Enveloping two objects by rolling contacts

are motivations to start this work.

By observing a series of motions shown in Fig.1, we can roughly divide the grasp into two types. Fig.1(b) shows a kind of inner-link based grasp where one finger contacts with an object only at the end link. Although many fingers are needed to grasp objects firmly, we can expect that the freedom of manipulation will increase since a finger can exert an arbitrary contact force at the contact point. On the other hand, Fig.1(c) shows the final state of the grasp where one finger contacts with an object at multiple points. In such a grasp, although a robot hand can grasp objects firmly with a small number of fingers, the freedom to manipulate the objects is strongly limited since a finger cannot exert an arbitrary contact force at the contact points. Due to the large potentiality of object manipulation with the end link, we focus on the grasp style as shown in Fig.1(b).

When multiple objects are manipulated simultaneously by rolling contact at each contact point, we have to take note of two factors. One is the object motion constraint. For the manipulation of two objects as shown in Fig.1(b), even if the left-hand object moves in an arbitrary direction, the right-hand object has to maintain the contact with the left-hand object and, therefore, cannot move arbitrarily. The other factor is

the dependency of contact force. Suppose that each finger exerts contact force onto the objects, as shown in Fig.1(b). Even if an arbitrary force (f_1 and f_2) is exerted at the contact point between the finger link and the object, the contact force (f_0) at the contact point between the two objects depends on both f_1 and f_2. Therefore, we have to consider the dependency of the contact force as well as the constraint on the object motion when manipulating multiple objects.

In this paper, we discuss the manipulation of multiple objects under rolling contacts by taking the above two issues into account. As for the discussion of the object motion constraint, we first formulate the motion constraint of the grasp, and show a necessary condition for manipulating multiple objects uniquely. As for the dependency of the contact force, we formulate the equation for computing the contact force among objects for a given set of finger forces by considering the dynamics of the system, and provide a condition for determining the contact force among objects uniquely. Under the condition for generating unique contact force among objects, we show a sufficient condition which ensures that each object can generate an arbitrary linear and rotational acceleration under the object motion constraint. Under the unique contact force among objects and the motion constraint caused by multiple contacts, we show a control scheme for the manipulation of multiple objects. Trajectory tracking of the object motion is experimentally performed by using a three-fingered hand to verify our idea. We believe that this experiment is the first attempt at making multiple objects follow along desired trajectories.

2 Relevant Work

Dauchez et al.[10] and Kosuge et al.[11] used two manipulators holding two objects independently and tried to apply to an assembly task. Aiyama et al.[12] studied a scheme for grasping multiple box type objects stably by using two manipulators. For an assembly task, Mattikalli et al.[13] proposed a method to find a stable alignments of multiple objects under the gravitational field. While these works treated multiple objects, they have considered neither the motion of the objects within the hand nor any manipulation of objects based on rolling contacts. The authors[9] have first studied the enveloping grasp for multiple objects. They have shown a condition to judge the rolling contact at each contact point and showed the rolling up condition.

Kerr et al.[2] and Montana[14] formulated the kinematics for manipulation of an object under rolling contacts with the fingertip. Li et al.[15] proposed a motion planning method with nonholonomic constraint. Howard et al.[1] and Maekawa et al.[16] studied the stiffness effect for the object motion with rolling. Cole et al.[5], Sarkar et al.[6], Bicchi et al[7] and Han et al.[8] proposed a control scheme for the object motion.

While there have been a number of works concerning the grasp and manipulation under rolling contacts, we believe that this is the first work for discussing the manipulation of multiple objects under rolling contacts.

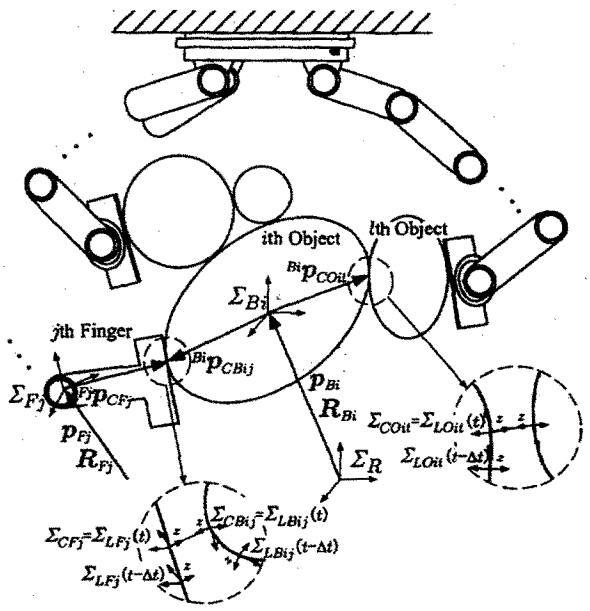

Fig. 2: Model of the system

3 Object Motion Constraint

Fig.2 shows the grasp of m objects by n fingers, where the finger j contacts with the object i, and additionally the object i has a common contact point with the object l. We assume the rolling contact at each contact point. Let Σ_R, Σ_{Bi} $(i = 1, \cdots, m)$ and Σ_{Fj} $(j = 1, \cdots, n)$ be the coordinate frames fixed at the base, at the center of gravity of the object i, and at the end link of the finger j, respectively. Σ_{CFj} and Σ_{CBij} denote the contact coordinate frames whose origins are always at the point of contact. Σ_{LFj} and Σ_{LBij} denote the local coordinate frames fixed relative to Σ_{Fj} and Σ_{Bi}, respectively, which coincide with Σ_{CFj} and Σ_{CBij} at time t, respectively. Σ_{COit} and Σ_{LOit} denote the contact and the local frame between objects, respectively$(t = 1, \cdots, r)$. Let $\boldsymbol{p}_* \in R^3$ and $\boldsymbol{R}_* \in R^{3\times3}$ be the position vector and the rotation matrix of the coordinate frame Σ_*, respectively. Let ${}^{*1}\boldsymbol{p}_{*2} \in R^3$ be the position vector of Σ_{*2} with respect to Σ_{*1}. We assume that the fingers have enough degrees of freedom to exert an arbitrary contact force and an arbitrary moment around the contact normal $(s_j \geq 4)$ where s_j denotes the number of joints of the finger j.

If we assume the pure rolling at each contact point, the linear velocity of the object i should coincide with the linear velocity of the finger j at the contact point, and the rotational velocity of the object i relative to the finger j about the contact normal should equal zero. Same discussion can be applied for the contact between the object i and the object l. These relationships are expressed as follows:

$$\boldsymbol{D}_{Bij} \begin{bmatrix} \dot{\boldsymbol{p}}_{Bi} \\ \boldsymbol{\omega}_{Bi} \end{bmatrix} = \boldsymbol{D}_{Fj} \begin{bmatrix} \dot{\boldsymbol{p}}_{Fj} \\ \boldsymbol{\omega}_{Fj} \end{bmatrix}, \qquad (1)$$

$$\boldsymbol{D}_{Oit}\begin{bmatrix}\dot{\boldsymbol{p}}_{Bi}\\ \boldsymbol{\omega}_{Bi}\end{bmatrix} = \boldsymbol{D}_{Olt}\begin{bmatrix}\dot{\boldsymbol{p}}_{Bl}\\ \boldsymbol{\omega}_{Bl}\end{bmatrix}, \tag{2}$$

$$\boldsymbol{D}_{Bij} = \begin{bmatrix}\boldsymbol{I}_3 & -((\boldsymbol{R}_{Bi}{}^{Bi}\boldsymbol{p}_{CBij})\times)\\ \boldsymbol{o} & \boldsymbol{e}_3^T\boldsymbol{R}_{LBij}^T\end{bmatrix} \in R^{4\times 6},$$

$$\boldsymbol{D}_{Fj} = \begin{bmatrix}\boldsymbol{I}_3 & -((\boldsymbol{R}_{Fj}{}^{Fj}\boldsymbol{p}_{CFj})\times)\\ \boldsymbol{o} & \boldsymbol{e}_3^T\boldsymbol{R}_{LFj}^T\end{bmatrix} \in R^{4\times 6},$$

$$\boldsymbol{D}_{Oit} = \begin{bmatrix}\boldsymbol{I}_3 & -((\boldsymbol{R}_{Bi}{}^{Bi}\boldsymbol{p}_{COit})\times)\\ \boldsymbol{o} & \boldsymbol{e}_3^T\boldsymbol{R}_{LOit}^T\end{bmatrix} \in R^{4\times 6},$$

where $\boldsymbol{I}_3$ denotes the 3×3 identity matrix, $(*\times)$ denotes the skew-symmetric matrix equivalent to the vector product, $\boldsymbol{\omega}_{Bi}$ and $\boldsymbol{\omega}_{Fj}$ denote the rotational velocity vectors of Σ_{Bi} and Σ_{Fj} with respect to Σ_R, respectively, and $\boldsymbol{e}_3 = [0\ 0\ 1]^T$. Aggregating eqs.(1) and (2) for $j = 1,\cdots,n$ and $t = 1,\cdots,r$, respectively, the equation of motion constraint is derived as follows:

$$\boldsymbol{D}_L\dot{\boldsymbol{p}}_{LF} = \boldsymbol{D}_B\dot{\boldsymbol{p}}_B, \tag{3}$$

where $\dot{\boldsymbol{p}}_{LF} = [\dot{\boldsymbol{p}}_{LF1}^T\ \boldsymbol{\omega}_{LF1}\cdots\dot{\boldsymbol{p}}_{LFn}^T\ \boldsymbol{\omega}_{LFn}]^T \in R^{4n}$, $\begin{bmatrix}\dot{\boldsymbol{p}}_{LFj}\\ \boldsymbol{\omega}_{LFj}\end{bmatrix} = \boldsymbol{D}_{Fj}\begin{bmatrix}\dot{\boldsymbol{p}}_{Fj}\\ \boldsymbol{\omega}_{Fj}\end{bmatrix} \in R^4$, $\dot{\boldsymbol{p}}_B = [\dot{\boldsymbol{p}}_{B1}^T\ \boldsymbol{\omega}_{B1}^T\ \cdots\ \dot{\boldsymbol{p}}_{Bm}^T\ \boldsymbol{\omega}_{Bm}^T]^T \in R^{6m}$, $\boldsymbol{D}_L = [\boldsymbol{I}_{4n}\ \boldsymbol{o}]^T \in R^{(4n+4r)\times 4n}$, $\boldsymbol{D}_B = [\boldsymbol{D}_{LB}^T\ \boldsymbol{D}_O^T]^T \in R^{(4n+4r)\times 6m}$, $\boldsymbol{D}_{LB} \in R^{4n\times 6m}$ includes $\boldsymbol{D}_{Bij}$, and $\boldsymbol{D}_O \in R^{4r\times 6m}$ includes both $\boldsymbol{D}_{Oit}$ and $\boldsymbol{D}_{Olt}$. In eq.(3), the matrix $\boldsymbol{D}_B$ is a function of both $\boldsymbol{p}_B$ and vectors ${}^{B1}\boldsymbol{p}_{CB11},\cdots$, and ${}^{Bi}\boldsymbol{p}_{COir}$. ${}^{B1}\boldsymbol{p}_{CB11},\cdots$, and ${}^{Bi}\boldsymbol{p}_{COir}$ are derived by utilizing the method proposed by Montana[14].

Now we consider the object motion constraint. For a grasp composed of multiple objects, the objects cannot move in an arbitrary direction due to eq.(3). Since $[-\boldsymbol{D}_L\ \boldsymbol{D}_B] \in R^{(4n+4r)\times(4n+6m)}$, the dimension of the solution of eq.(3) depends on $4n+6m-\mathrm{rank}[-\boldsymbol{D}_L\ \boldsymbol{D}_B]$. Moreover, $4n+6m-\mathrm{rank}[-\boldsymbol{D}_L\ \boldsymbol{D}_B] = 6m-\mathrm{rank}\boldsymbol{D}_O$ is always satisfied. Based on these discussions, we now define the dimension of object motion as follows:

Definition 1 (Dimension of object motion) *For the grasp of multiple objects, the grasped objects have*

$$\begin{aligned} I_M &= 4n+6m-\mathrm{rank}[-\boldsymbol{D}_L\ \boldsymbol{D}_B]\\ &= 6m-\mathrm{rank}\boldsymbol{D}_O \end{aligned} \tag{4}$$

dimensional motion.

It should be noted that, since the terms with respect to finger motion ($4n$ and $\boldsymbol{D}_L$) disappear in the second row of eq.(4), I_M shows the pure dimension of the motion of grasped objects.

We now introduce a new vector $\boldsymbol{\zeta} \in R^{I_M}$ whose dimension is same as that of object motion. It should be noted that $\boldsymbol{\zeta}$ includes the independent variables controlling the motion of the grasped objects. Now, let us define $\boldsymbol{\zeta}$ as $\dot{\boldsymbol{\zeta}} = \boldsymbol{E}_L\dot{\boldsymbol{p}}_{LF} + \boldsymbol{E}_B\dot{\boldsymbol{p}}_B \in R^{I_M}$, where the matrices $\boldsymbol{E}_L$ and $\boldsymbol{E}_B$ are defined in such a way that these matrices have the minimum size making $\begin{bmatrix}-\boldsymbol{D}_L & \boldsymbol{D}_B\\ \boldsymbol{E}_L & \boldsymbol{E}_B\end{bmatrix}$ full column rank in the following equation:

$$\begin{bmatrix}-\boldsymbol{D}_L & \boldsymbol{D}_B\\ \boldsymbol{E}_L & \boldsymbol{E}_B\end{bmatrix}\begin{bmatrix}\dot{\boldsymbol{p}}_{LF}\\ \dot{\boldsymbol{p}}_B\end{bmatrix} = \begin{bmatrix}\boldsymbol{o}\\ \dot{\boldsymbol{\zeta}}\end{bmatrix}. \tag{5}$$

We note that, since $\boldsymbol{D}_L$ is composed of $\boldsymbol{I}_{4n}$ in the upper side, we can always make the above matrix full column rank even when $\boldsymbol{E}_L = \boldsymbol{o}$. Assuming $\boldsymbol{E}_L = \boldsymbol{o}$, since $\dot{\boldsymbol{\zeta}}$ becomes a function of $\dot{\boldsymbol{p}}_B$ as $\dot{\boldsymbol{\zeta}} = \boldsymbol{E}_B\dot{\boldsymbol{p}}_B$, $\boldsymbol{\zeta}$ can express the motion of the grasped objects. We also note that the selection of $\boldsymbol{\zeta}$ is not unique.

By solving eq.(5) with respect to $\dot{\boldsymbol{p}}_{LF}$ and $\dot{\boldsymbol{p}}_B$, the following equation is derived:

$$\begin{aligned}\begin{bmatrix}\dot{\boldsymbol{p}}_{LF}\\ \dot{\boldsymbol{p}}_B\end{bmatrix} &= \begin{bmatrix}-\boldsymbol{D}_L & \boldsymbol{D}_B\\ \boldsymbol{E}_L & \boldsymbol{E}_B\end{bmatrix}^+\begin{bmatrix}\boldsymbol{o}\\ \dot{\boldsymbol{\zeta}}\end{bmatrix}\\ &= \begin{bmatrix}\boldsymbol{L}\\ \boldsymbol{B}\end{bmatrix}\dot{\boldsymbol{\zeta}}, \end{aligned} \tag{6}$$

where $*^+$ denotes the pseudo-inverse of $*$. It should be noted that, since $\begin{bmatrix}-\boldsymbol{D}_L & \boldsymbol{D}_B\\ \boldsymbol{E}_L & \boldsymbol{E}_B\end{bmatrix}$ is full column rank, the null space does not exist.

To have the unique object motion, $\dot{\boldsymbol{\zeta}}$ has to be also uniquely determined for a given finger tip motion $\dot{\boldsymbol{p}}_{LF}$; otherwise there exists an arbitrary motion for at least one object even when the finger motion is determined. By using eq.(6), the unique determination of $\dot{\boldsymbol{\zeta}}$ is guaranteed by the following condition:

Condition 1 (Kinematic condition for manipulation) *A necessary condition for a robot hand to uniquely determine the object motion is*

$$\mathrm{Ker}(\boldsymbol{L}) = \emptyset, \tag{7}$$

where $\mathrm{Ker}(*)$ *denotes the null space of* $*$.

For a single object, $\mathrm{Ker}(\boldsymbol{L}) = \emptyset$ when a three-fingered hand grasps an object, while $\mathrm{Ker}(\boldsymbol{L}) \neq \emptyset$ when an object is simply placed on a palm or a table. Condition 1 (Kinematic condition for manipulation) is a necessary condition since we do not consider the contact force applied to the objects. Since $\mathrm{size}\boldsymbol{L} = 4n\times(6m-\mathrm{rank}\boldsymbol{D}_O)$ [1], this condition can be finally achieved when the number of fingers increases.

4 Dependency of Contact Force

4.1 Equation of Dependency

We now make clear the dependency of contact force among objects. Since the contact force and moment are applied to the objects by the finger links, the

[1] To express the size of a matrix, we define a function $\mathrm{size}A = m\times n$ for a matrix $A \in R^{m\times n}$.

equation of motion of the grasped objects is given by

$$M_B\ddot{p}_B + h_B = D_{LB}^T f_{CB} + D_O^T f_{CO}, \quad (8)$$

$$f_{CB} = [f_{CB1}^T\ n_{CB1}\ \cdots f_{CBn}^T\ n_{CBn}]^T \in R^{4n},$$
$$f_{CO} = [f_{CO1}^T\ n_{CO1}\ \cdots f_{COr}^T\ n_{COr}]^T \in R^{4r},$$

where f_{CBj} and n_{CBj} $(j = 1, \cdots, n)$ denote the contact force and the moment about the contact normal applied by the finger j, respectively, and f_{COt} and n_{COt} $(t = 1, \cdots, r)$ denote the contact force and the moment about the contact normal at the t-th contact between objects, respectively, where we assume that the object l can apply the contact force to the object i when $i < l$. M_B and h_B denote the inertia matrix and the vector with respect to the centrifugal and the Coriolis' force, respectively. From eq.(3), the constraint condition among objects is expressed as

$$D_O\dot{p}_B = o. \quad (9)$$

By using eq.(8) and the differentiation of eq.(9), the following relation is derived:

$$Af_C = b, \quad (10)$$

where $A = [D_O M_B^{-1} D_{LB}^T \quad D_O M_B^{-1} D_O^T]$ and $b = D_O M_B^{-1} h_B - \dot{D}_O \dot{p}_B$. Eq.(10) shows the dependency of the contact force, namely f_O is dependent on f_{CB}.

4.2 Uniqueness of Contact Force

Now, we have a couple of questions. Can we always find the unique contact force f_{CO}? If this is not the case, under what condition can we have the unique f_{CO}? Let us now answer these questions. If $D_O M_B^{-1} D_O^T$ in eq.(10) is nonsingular, f_{CO} can be expressed uniquely in the following form:

$$f_{CO} = (D_O M_B^{-1} D_O^T)^{-1}(-D_O M_B^{-1} D_{LB}^T f_{CB} + D_O M_B^{-1} h_B - \dot{D}_O \dot{p}_B). \quad (11)$$

Therefore, the nonsingularity of $D_O M_B^{-1} D_O^T$ is the necessary condition for finding the unique f_{CO}. Taking the nonsingularity of M_B into account, the nonsingularity of $D_O M_B^{-1} D_O^T$ is equivalent to the following condition:

Condition 2 (Uniqueness of contact force) *The necessary condition for the contact force among objects to be uniquely determined is given by*

$$\mathrm{rank} D_O = 4r, \quad (12)$$

This condition can be satisfied under $4r \leq 6m$ if we assume that D_O is a full rank matrix.

Now, suppose that a contact force among objects is not uniquely determined. Under such a condition, it is not ensured whether or not the contact force always exists within the friction cone at the point of contact. The contact force components in the null space produce a slipping motion at the contact point.

4.3 Internal Forces

Now, we discuss the relationship between the dependency of the contact force and the internal force. By the relationship of duality between force and infinitesimal displacement, we can obtain the force balance equation for multiple objects as follows:

$$f_B = D_B^T f_C = D_{LB}^T f_{CB} + D_O^T f_O, \quad (13)$$

$$f_B = [f_{B1}^T\ n_{B1}^T\ \cdots\ f_{Bm}^T\ n_{Bm}^T]^T \in R^{6m},$$

where f_{Bi} and n_{Bi} $(i = 1, \cdots, m)$ denote the force and the moment at the center of gravity of the object i, respectively.

Since $D_B \in R^{(4n+4r)\times 6m}$, a homogeneous solution exists, which means that there exists internal force according to $4n + 4r - \mathrm{rank} D_B^T$. Same discussion can be applied for $D_O \in R^{4r\times 6m}$. Here, we define the dimensions of the internal forces as follows:

Definition 2 (Dimension of internal force) *For a grasp of multiple objects, the grasped objects have the following internal forces*

$$I_I = 4n + 4r - \mathrm{rank} D_B^T, \quad (14)$$
$$I_{IO} = 4r - \mathrm{rank} D_O^T, \quad (15)$$

where I_I and I_{IO} denote the dimension of the total internal force and the internal force among objects, respectively.

For the grasp satisfying $I_{IO} > 0$, the internal force occurs passively even if all the contacts with fingers are released and if all the contacts among objects are maintained. This internal force among objects is peculiar to multiple objects and is not affected by the finger forces. By comparing Definition 2 (dimension of internal force) with Condition 2 (uniqueness of contact force), since we can see that Condition 2 is the same as $I_{IO} = 0$, the contact force can be determined uniquely when the internal force among objects does not exist. Moreover, since $\mathrm{rank} D_B \leq \mathrm{rank} D_{LB} + \mathrm{rank} D_O$ is satisfied, the internal force among objects is included in the total internal force. Therefore, we cannot apply an arbitrary internal force for the grasp without satisfying Condition 2 (uniqueness of contact force).

5 A Sufficient Condition for Manipulation

We now examine a sufficient condition for robot hands to manipulate the grasped objects arbitrarily under the motion constraint while maintaining the friction constraint.

Each contact force should exist within the friction cone as long as each contact avoids a slipping motion. Approximating the friction cone by the h faced polyhedral convex cone, we can express each contact force and moment as follows:

$$f_C = V\lambda, \qquad \lambda \geq o, \quad (16)$$

We assume that Condition 2 is satisfied. Different from the condition for the manipulation of a single object, we have to take the motion constraint and the dependency of the contact force into account. For an arbitrary manipulation, we have to examine whether the objects can generate an arbitrary acceleration or not[3]. By using eqs.(6) and (8), the following equation is derived:

$$\boldsymbol{M}_B B\ddot{\boldsymbol{\zeta}} = \boldsymbol{D}_B^T \boldsymbol{f}_C - \hat{\boldsymbol{h}}_B, \tag{17}$$

where $\hat{\boldsymbol{h}}_B = \boldsymbol{h}_B + \boldsymbol{M}_B \dot{B}\dot{\boldsymbol{\zeta}}$. Eq.(17) shows the relationship between the object acceleration and the contact force under the motion constraint among objects. Since the dependency of contact force is not taken into consideration in eq.(17), we consider eq.(10) and formulate the linear programming problem. To make $\boldsymbol{AV}_1$ nonsingular, we partition $\boldsymbol{V}$ and $\boldsymbol{\lambda}$ as $\boldsymbol{V} = [\boldsymbol{V}_1\ \boldsymbol{V}_2]$ and $\boldsymbol{\lambda} = [\boldsymbol{\lambda}_1^T\ \boldsymbol{\lambda}_2^T]^T$, respectively. Since $\boldsymbol{A}$ is full row rank under Condition 2, we can always make $\boldsymbol{AV}_1$ nonsingular by a proper method of partition. By using this partition, eq. (10) is rewritten as follows:

$$\boldsymbol{\lambda}_1 = -(\boldsymbol{AV}_1)^{-1}(\boldsymbol{AV}_2\boldsymbol{\lambda}_2 - \boldsymbol{b}). \tag{18}$$

Substituting eq.(18) into eq.(17), we consider the following linear programming problem:

$$\begin{aligned} \text{Maximize} \quad & z = \min[\boldsymbol{\lambda}_1^T\ \hat{\boldsymbol{\lambda}}_1^T]^T, \\ \text{Subject to} \quad & \boldsymbol{M}_B B\ddot{\hat{\boldsymbol{\zeta}}} = \boldsymbol{H}\boldsymbol{\lambda}_2, \\ & \boldsymbol{\lambda}_2 \geq \mathbf{o}, \end{aligned} \tag{19}$$

where

$$\begin{aligned} \boldsymbol{M}_B B\ddot{\hat{\boldsymbol{\zeta}}} &= \boldsymbol{M}_B B\ddot{\boldsymbol{\zeta}} - \boldsymbol{D}_B^T \boldsymbol{V}_1(\boldsymbol{AV}_1)^{-1}\boldsymbol{b} - \hat{\boldsymbol{h}}_B, \\ \hat{\boldsymbol{\lambda}}_1 &= \boldsymbol{\lambda}_1 - (\boldsymbol{AV}_1)^{-1}\boldsymbol{b} \\ &= -(\boldsymbol{AV}_1)^{-1}\boldsymbol{AV}_2\boldsymbol{\lambda}_2, \\ \boldsymbol{H} &= \boldsymbol{D}_B^T\boldsymbol{V}_2 - \boldsymbol{D}_B^T\boldsymbol{V}_1(\boldsymbol{AV}_1)^{-1}\boldsymbol{AV}_2. \end{aligned}$$

Due to eq.(18), we cannot impose any constraints for $\boldsymbol{\lambda}_1$ such as $\boldsymbol{\lambda}_1 \geq \mathbf{o}$. Therefore, we consider maximizing the minimum element of $\boldsymbol{\lambda}_1$ in the objective function. Moreover, in eqs.(17) and (18), there are nonlinear terms with respect to the centrifugal and Colioris' force, $\hat{\boldsymbol{h}}_B$ and $\boldsymbol{b}$. To get rid of these terms from the formulation, we used the coordinate transformation from $\boldsymbol{\zeta}$ to $\hat{\boldsymbol{\zeta}}$.

We now derive a condition for generating arbitrary acceleration for multiple objects. Let $\boldsymbol{e}_1, \boldsymbol{e}_2, \cdots, \boldsymbol{e}_{I_M} \in R^{I_M}$ be I_M number of given linearly independent vectors[3]. Let $\boldsymbol{\lambda}_1 = \boldsymbol{\lambda}_{1+1}, \cdots, \boldsymbol{\lambda}_{1+I_M}, \boldsymbol{\lambda}_{1-1}, \cdots, \boldsymbol{\lambda}_{1-I_M}$, $\hat{\boldsymbol{\lambda}}_1 = \hat{\boldsymbol{\lambda}}_{1+1}, \cdots, \hat{\boldsymbol{\lambda}}_{1+I_M}, \hat{\boldsymbol{\lambda}}_{1-1}, \cdots, \hat{\boldsymbol{\lambda}}_{1-I_M}$, and $\boldsymbol{\lambda}_2 = \boldsymbol{\lambda}_{2+1}, \cdots, \boldsymbol{\lambda}_{2+I_M}, \boldsymbol{\lambda}_{2-1}, \cdots, \boldsymbol{\lambda}_{2-I_M}$ be the solutions of the linear programming problem (19) for $\ddot{\hat{\boldsymbol{\zeta}}} = \boldsymbol{e}_1, \cdots, \boldsymbol{e}_{I_M}, -\boldsymbol{e}_1, \cdots, -\boldsymbol{e}_{I_M}$, respectively. Now we have the following condition:

Condition 3 (Generation of arbitrary acceleration) *Assume that Condition 2 is satisfied. A sufficient condition for the grasped objects to generate the acceleration $\ddot{\boldsymbol{\zeta}}$ arbitrarily is that the linear programming problem (19) has solutions for $2I_M$ number of $\ddot{\hat{\boldsymbol{\zeta}}} = \pm\boldsymbol{e}_k$, $(k = 1, \cdots, I_M)$ satisfying both $\boldsymbol{\lambda}_{1\pm k} \geq \mathbf{o}$ for all $\boldsymbol{\lambda}_{1\pm k}$ $(k = 1, \cdots, I_M)$ and $\hat{\boldsymbol{\lambda}}_{1+k} + \hat{\boldsymbol{\lambda}}_{1-k} > \mathbf{o}$ for at least one pair of $\hat{\boldsymbol{\lambda}}_{1+k}$ and $\hat{\boldsymbol{\lambda}}_{1-k}$ $(k = 1, \cdots, I_M)$.*

Proof See Appendix.

To solve the linear programming problem, we can use a set of the orthonormal basis as $\boldsymbol{e}_k$ $(k = 1, \cdots, I_M)$. Condition 3 (generation of arbitrary acceleration) is a sufficient condition since we set the approximated friction cone existing inside of the actual friction cone. Diverging from the condition for the manipulation of a single object[3], we substituted vectors $\boldsymbol{e}_k$ $(k = 1, \cdots, I_M)$ into the acceleration $\ddot{\hat{\boldsymbol{\zeta}}}$ taking the motion constraints into consideration. Moreover, due to the term $\boldsymbol{b}$ in eq.(18), the condition $\hat{\boldsymbol{\lambda}}_{1+k} + \hat{\boldsymbol{\lambda}}_{1-k} > \mathbf{o}$ is added for the reason shown in the Appendix. To satisfy $\hat{\boldsymbol{\lambda}}_{1+k} + \hat{\boldsymbol{\lambda}}_{1-k} > \mathbf{o}$, we consider maximizing the minimum of $\hat{\boldsymbol{\lambda}}_1$ in the objective function.

So far, we have shown three conditions for manipulating multiple objects, i.e., the kinematic condition (Condition 1), the uniqueness of contact force among objects (Condition 2), and generation of arbitrary acceleration (Condition 3). Since Condition 3 is a sufficient condition for manipulation, Condition 1 should be included in Condition 3.

6 Controller

We now derive a controller for the object motion. We first discuss the relationship between the end link and the joints of each finger, so that we may introduce the control scheme in joint level. Since the velocity at the end link of each finger can be expressed by the joint velocity of each finger, we obtain the following relationship:

$$\boldsymbol{J}_F\dot{\boldsymbol{\theta}} = \boldsymbol{D}_{LB}\dot{\boldsymbol{p}}_B. \tag{20}$$

The equation of the finger motion is derived by using the Lagrange's method as follows:

$$\boldsymbol{M}_F\ddot{\boldsymbol{\theta}} + \boldsymbol{h}_F = \boldsymbol{\tau} - \boldsymbol{J}_F^T\boldsymbol{f}_{CB}, \tag{21}$$

where $\boldsymbol{M}_F$, $\boldsymbol{h}_F$ and $\boldsymbol{\tau}$ are the inertia matrix, the vector with respect to the centrifugal and Coriolis' force and the joint torque vector, respectively.

As a controller for object manipulation, we extended the trajectory controller proposed for the grasp of a single object[17]. Since the derivation of the controller is almost same as that which is described in [17], we omit the formulation in detail. Assuming that each finger does not have the redundant degrees of freedom $(s_j = 4, j = 1, \cdots, n)$, this controller has the following form as a joint torque command:

$$\boldsymbol{\tau} = \boldsymbol{J}_F^T(\boldsymbol{B}^T\boldsymbol{D}_{LB}^T)^+\boldsymbol{F} + \boldsymbol{J}_F^T\boldsymbol{N}_B\boldsymbol{k}_B, \tag{22}$$

$$\boldsymbol{F} = \bar{\boldsymbol{M}}_B(\ddot{\boldsymbol{\zeta}}_d - \boldsymbol{K}_V\dot{\boldsymbol{\delta}}_\zeta - \boldsymbol{K}_P\boldsymbol{\delta}_\zeta) + \bar{\boldsymbol{h}}_B, \tag{23}$$

where

$$\bar{M}_B = B^T(M_B + D_{LB}^T J_F^{-T} M_F J_F^{-1} D_{LB})B,$$

$$\bar{h}_B = B^T h_B + B^T D_{LB}^T J_F^{-T}(M_F \frac{d}{dt}(J_F^{-1} D_{LB})\dot{p}_B + h_F)$$

$$\delta_\zeta = \zeta - \zeta_d,$$

K_P and K_V are the diagonal matrices corresponding to feedback gain, and ζ_d is the desired value of ζ. N_B is the null space of $B^T D_{LB}^T$ where $B^T D_{LB}^T N_B = \mathbf{o}$ is satisfied. The first term of the right-hand side of eq.(22) controls the position of the objects to the desired trajectory, and the second term controls the internal force to the desired value.

Here, the control law itself does not ensure that the contact force caused by both finger link and other objects produces an arbitrary acceleration for the parameter ζ. Therefore, before applying the control law, we have to confirm whether an arbitrary acceleration can be achieved or not based on Condition 3 (generation of arbitrary acceleration). Moreover, to confirm whether the desired internal force can be realized or not, the grasp must satisfy Condition 2 (uniqueness of contact force).

7 Examples

For simplicity, we consider 2D examples. Fig.3 shows the grasps used for numerical examples, where the results are shown in Table 1.

For the grasp as shown in Fig.3(a), the grasped objects have four dimensional motion and zero dimensional internal force since $I_M = 4$ and $I_I = 0$. The physical interpretation of this dimension of motion is shown in Fig.4 which depicts two dimensions for translational motion at the center of gravity between two objects(Fig.4(a) and (b)), one dimension for rotational motion around the center of gravity (Fig.4(c)), and one dimension for rotational motion without changing the rotation angle around the center of mass (Fig.4(d)). The grasp shown in Fig.3(a) satisfies Condition 1 (kinematic condition for manipulation) because the finger can manipulate the objects arbitrarily under the motion constraint if we assume that two objects always make contact at each contact point. Condition 2 (uniqueness of contact force, rank$D_O = 2r$ for 2D example) is also satisfied. On the other hand, Condition 3 (generation of arbitrary acceleration) is not satisfied because the contact force does not exist when two objects move in a downward direction.

Fig.3(b) also shows the grasp of two objects by two fingers. The difference between Fig.3(a) and Fig.3(b) is the position of the contact point between a finger and an object. This configuration satisfies neither Condition 1 nor 3, which means that two objects can move freely in a vertical direction even if two finger positions are fixed. Therefore, the fingers cannot control the objects' motion in a vertical direction.

The grasp as shown in Fig.3(c) has four dimensional motion and four dimensional total internal force since $I_M = 4$ and $I_I = 4$. The physical interpretation of this dimension of the total internal force is shown in Fig.5. In this grasp configuration, the force focus exists for both objects, and these force focuses lie on the line including the contact point between the two objects. Thus, we have two dimensions for two force focuses capable of moving on the line(Fig.5(a) and (b)), and have one dimension for the rotation of the line(Fig.5(c)). The other one dimension is the magnitude of the internal force with the position of the force focuses unchanged(Fig.5(d)).

The grasp of four objects as shown in Fig.3(d) has five contact points among objects. This grasp configuration has three dimensional motion ($I_M = 3$) and six dimensional total internal force ($I_I = 6$). Moreover, the total internal force contains one dimensional internal force among objects($I_{IO} = 1$). Therefore, since $I_{IO} \neq 0$, Condition 2 (uniqueness of contact force) is not satisfied. Since AV_1 is not invertible, eq.(19) cannot be formulated, and we cannot judge Condition 3 (generation of arbitrary acceleration).

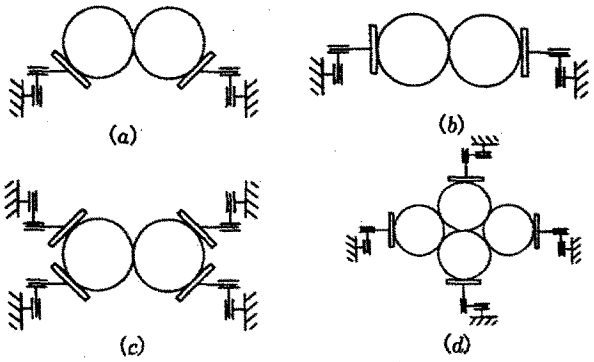

Fig. 3: Several grasp configurations

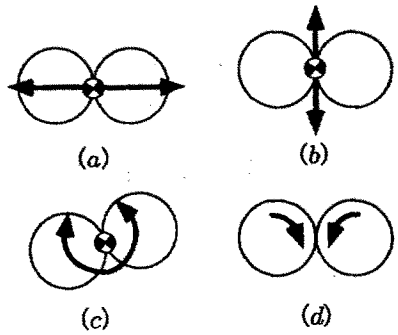

Fig. 4: Four motion degrees of freedom

8 Experiment

We performed the experiment of trajectory tracking by using the Hiroshima Hand[18]. The Hiroshima Hand is composed of three planar finger units, where each finger has the same structure and has three joints. Since the Hiroshima Hand is driven by the velocity servo, we used the following controller as a joint velocity

Table 1: Result of calculation

	sizeD_B	rankD_B	size $\begin{bmatrix} -D_L^T \\ D_B^T \end{bmatrix}$	rank $\begin{bmatrix} -D_L^T \\ D_B^T \end{bmatrix}$	I_M	I_I	I_{IO}	Cond.1	Cond.2	Cond.3
(a)	6×6	6	6×10	6	4	0	0	○	○	×
(b)	6×6	5	6×10	6	4	1	0	×	○	×
(c)	10×6	6	10×14	10	4	4	0	○	○	○
(d)	18×12	12	18×20	17	3	6	1	○	×	

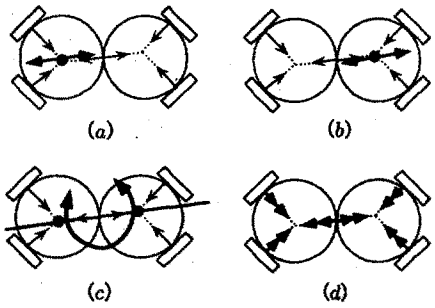

Fig. 5: Four internal force degrees of freedom

command:

$$\dot{\theta} = KJ_F^+C(\zeta_d - \zeta), \qquad (24)$$

where K is the gain matrix. For a planar grasp, each finger must have at least two joints. Since each finger has three joints, Pseudo-inverse of J_F is used in the command. The radius of each object is 0.01[m]. The objects contact with the third link of each finger. In the experiment, x_{B1}, y_{B1} and ϕ_{B1} are planned to keep their initial values while y_{B2} increases 0.01[m] in 1 [sec]. The position of the objects are measured by analyzing the image taken in a video tape. From Fig.6, we can see that the objects fairly well follow along the desired trajectory.

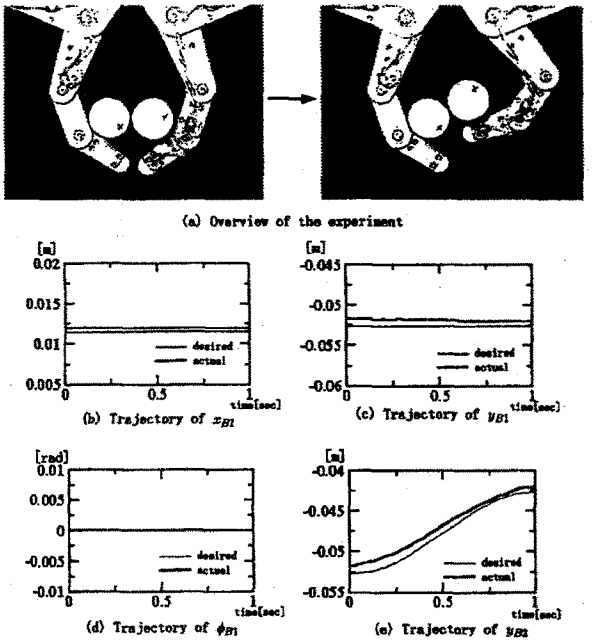

Fig. 6: Experimental results

9 Conclusions

This paper discussed the manipulation of multiple objects under rolling contacts. For manipulating multiple objects, each object's motion is restricted by the other objects, and the contact force between objects is not controlled independently. Taking the above issues into account, we showed three conditions for the manipulation of multiple objects, i.e., a kinematic condition for determining object motion uniquely, a sufficient condition for generating arbitrary acceleration on the objects, and the necessary and sufficient condition for determining the contact force among objects uniquely.

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A Proof for Condition 3

If the linear programming problem (19) has solutions for $2I_M$ number of $\ddot{\hat{\zeta}} = \pm e_k$, $(k = 1, \cdots, I_M)$, we have the following $2I_M$ number of equations:

$$\begin{aligned} M_B B e_1 &= H\lambda_{2+1}, \\ &\vdots \\ M_B B e_{I_M} &= H\lambda_{2+I_M}, \\ -M_B B e_1 &= H\lambda_{2-1}, \\ &\vdots \\ -M_B B e_{I_M} &= H\lambda_{2-I_M}. \end{aligned}$$

By introducing non-negative scolors $\rho_{+1}, \cdots, \rho_{+I_M}$, $\rho_{-1}, \cdots, \rho_{-I_M}$, an arbitrary $\ddot{\hat{\zeta}}$ can be expressed as

$$\begin{aligned} \ddot{\hat{\zeta}} &= \rho_{+1}e_1 + \cdots + \rho_{+I_M}e_{I_M} - \rho_{-1}e_1 - \cdots - \rho_{-I_M}e_{I_M} \\ &= (\rho_{+1} - \rho_{-1})e_1 + \cdots + (\rho_{+I_M} - \rho_{-I_M})e_{I_M}. \end{aligned} \quad (25)$$

Corresponding to eq.(25), the solutions for λ_2 are expressed as

$$\begin{aligned} \lambda_2 &= \rho_{+1}\lambda_{2+1} + \cdots + \rho_{+I_M}\lambda_{2+I_M} \\ &\quad + \rho_{-1}\lambda_{2-1} + \cdots + \rho_{-I_M}\lambda_{2-I_M} \geq \mathbf{o}. \end{aligned} \quad (26)$$

Thus we obtain λ_2 generating arbitrary $\ddot{\hat{\zeta}}$. We now examine whether we can always make $\lambda_1 \geq \mathbf{o}$ for arbitrary $\ddot{\hat{\zeta}}$. From eq.(18), the following equations are derived:

$$\begin{aligned} \lambda_{1+1} &= -(AV_1)^{-1}(AV_2\lambda_{2+1} - b), \\ &\vdots \\ \lambda_{1+I_M} &= -(AV_1)^{-1}(AV_2\lambda_{2+I_M} - b), \\ \lambda_{1-1} &= -(AV_1)^{-1}(AV_2\lambda_{2-1} - b), \\ &\vdots \\ \lambda_{1-I_M} &= -(AV_1)^{-1}(AV_2\lambda_{2-I_M} - b). \end{aligned} \quad (27)$$

By summing all equations in eqs.(27), the following equation is derived:

$$\begin{aligned} &\rho_{+1}\hat{\lambda}_{1+1} + \cdots + \rho_{-I_M}\hat{\lambda}_{1-I_M} + (AV_1)^{-1}b = \\ &-(AV_1)^{-1}\{AV_2(\rho_{+1}\lambda_{2+1} + \cdots + \rho_{-I_M}\lambda_{2-I_M}) - b\} \end{aligned} \quad (28)$$

Although the left hand side of eq.(28) expresses λ_1, we cannot always insure $\lambda_1 \geq \mathbf{o}$ since b is included in the left hand side of eq.(28). However, since $\hat{\lambda}_{1+k}$ and $\hat{\lambda}_{1-k}$ correspond to the acceleration in the opposite direction, $\ddot{\hat{\zeta}} = e_k$ and $\ddot{\hat{\zeta}} = -e_{-k}$, respectively, there is no effect of acceleration if we increase λ_1 in the direction of $\hat{\lambda}_{1+k} + \hat{\lambda}_{1-k}$. In other words, by observing eq.(25), we can put $\rho_{+k} = \hat{\rho}_{+1} + \alpha$ and $\rho_{-k} = \hat{\rho}_{-1} + \alpha$ for an arbitrary α without loss of the arbitrariness of $\ddot{\hat{\zeta}}$ since α disappears as $\rho_{+k} - \rho_{-k} = \hat{\rho}_{+k} - \hat{\rho}_{-k}$. Since we can set α large enough, we can always make $\lambda_1 = \rho_{+1}\hat{\lambda}_{1+1} + \cdots + \rho_{-I_M}\hat{\lambda}_{1-I_M} + (AV_1)^{-1}b \geq 0$ if $\hat{\lambda}_{1+k} + \hat{\lambda}_{1-k} > 0$ is satisfied. Now we can always satisfy $\lambda_1 \geq \mathbf{o}$ and $\lambda_2 \geq \mathbf{o}$ for an arbitrary $\ddot{\hat{\zeta}}$. Since $M_B B$ is always full column rank, the arbitrariness of $\ddot{\hat{\zeta}}$ is equivalent to the arbitrariness of $\ddot{\zeta}$. These discussions hold the theorem.