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Dynamic Control of Redundant Manipulators Using Artificial Potential Field Approach with Time Scaling

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Abstract

This paper proposes a new method for dynamic control of redundant manipulators via artificial potential field approach (APFA). The proposed method is based on the APFA with a combination of a time scale transformation and a time base generator which works as a time scale compressor and can control the dynamic behavior of the robot without any change of the form of the designed controller itself. The effectiveness of the proposed method is verified by computer simulations for a three-joint planar manipulator.

1 Introduction

In the Artificial Potential Field Approach (APFA) [1]-[4], the goal is represented by an artificial attractive potential field and the obstacles by corresponding repulsive fields, so that the trajectory to the target can be generated via a flow-line tracking process with consideration to the obstacle avoidance. This method is often used for the trajectory generation problem of robots because of its simplicity and lower computation than other methods using global information about the task space. However, little attention has been paid to control the dynamic behavior of the generated trajectories such as movement time from the initial position to the goal and velocity profile of the generated trajectory. Although one of the most crucial winning features of the APFA is real-time applicability, it is difficult to use the generated trajectory for the control of the robots in real time.

For the disadvantage of the APFA mentioned above, H. Hashimoto et al. [4] proposed a method using an electrostatic potential field and a sliding mode that can regulate the movement time but not the dynamic behavior of a robot. T. Tsuji et al. [5][6] proposed a method introducing the Time Base Generator (TBG) into the APFA which can regulate the movement time and also the velocity profile of the robot, but can not be applied to the dynamic control.

On the other hand, J.M.Hollerbach [7] developed the trajectory time-scaling method for the torque limited path following problem of the manipulator. The method can lead the end-effector to the goal along the given path by modifying the movement speed. M. Sampei and K. Furuta [8] showed that the stability of a system is preserved for any time scale transformation as long as the defined new time never goes backward against the actual time. More recently, Y. Tanaka et.al. [9] has developed the trajectory generation method for the dynamic control of robots based on the APFA with a combination of time scale transformation and TBG. Then, they applied it to the dynamic control of a holonomic mobile robot.

In this paper, we propose a new trajectory generation method for the dynamic control of redundant manipulators using Tanaka's method [9]. The redundant manipulator has a desirable feature that may lead to more dexterity and versatility of the robot motions, for instance, avoiding obstacles or singular configurations when performing a given task [10]-[12]. The present method can control the spatio-temporal trajectories of the end-effector with significant advantages of redundancy.

This paper is organized as follows: Section II formulates dynamics of a redundant manipulator. Section III points out the general problems of the APFA. Then, the new trajectory generation method based on the APFA is explained in detail in Section IV. Finally, the effectiveness of the proposed method is shown from computer simulations with the dynamic model of the redundant manipulator in Section V.

2 Dynamics of manipulators

The joint space motion equation of an n-degree-of-freedom manipulator whose end-effector is operating in the m-dimensional task space can be expressed as

$$M(q)\ddot{q} + h(q,\dot{q}) + g(q) = \tau\,, \qquad (1)$$

where $q \in \Re^n$ is the joint angle vector, $M(q) \in \Re^{n\times n}$ is the non-singular inertia matrix (hereafter denoted by M), $h(q, \dot{q}) \in \Re^n$ is the Coliolies and centrifugal force term, $g(q) \in \Re^n$ is the gravity term, and $\tau \in \Re^n$ is the joint torque vector. On the other hand, the dynamics of the end-effector can be written in the operational space as [12]

$$M_x(q)\ddot{x} + h_x(q, \dot{q}) + g_x(q) = F\ , \qquad (2)$$

where $x \in \Re^m$ is the current end-effector position, $F \in \Re^m$ is the end-effector force vector, $M_x(q) = (JM^{-1}J^T)^{-1} \in \Re^{m\times m}$ is the operational space kinetic energy matrix (hereafter denoted by M_x), $J \in \Re^{m\times n}$ is the Jacobian matrix, and also $h_x(q,\dot{q}) = \bar{J}^T h(q,\dot{q}) - M_x\dot{J}\dot{q}$, $g_x(q) = \bar{J}^T g(q)$, $\bar{J} = (M_x J M^{-1})^T$.

When a manipulator possesses extra degree-of-freedom to execute a given task, i.e. $m < n$, the joint torque of redundant manipulators can be decomposed into two elements; the joint torque $\tau_{effector} \in \Re^n$ to operate the end-effector, and the joint torque $\tau_{joint} \in \Re^n$ to control the additional freedom of joint motion with redundancy of a manipulator. The force/torque relationship between the joint torque $\tau_{effector}$ and the operational force F is given by

$$\tau_{effector} = J^T F\ . \qquad (3)$$

On the other hand, the joint torque τ_{joint} always satisfies the following condition [12] given by

$$\bar{J}^T \tau_{joint} = 0\ . \qquad (4)$$

This equation implies that the joint torque τ_{joint} must lay in the null space associated with the matrix $\bar{J}^T$ so as not to produce any acceleration at the end-effector. The general solution τ_{joint} for this condition given by

$$\tau_{joint} = G\tau^*\ , \qquad (5)$$

where τ^* is an arbitrary n-dimensional vector, and $G = I - J^T\bar{J}^T \in \Re^{n\times n}$. Consequently, the total joint torque τ for a redundant manipulator can be recomposed of (3) and (5) as follows:

$$\begin{aligned} \tau &= \tau_{effector} + \tau_{joint} \\ &= J^T F + G\tau^*\ . \end{aligned} \qquad (6)$$

In this paper, we design the feedback control law F and τ^*, respectively. The total joint torque composed of those designed controllers allows a redundant manipulator to perform a given task by utilizing arm redundancy efficiently.

3 Artificial Potential Field Approach

In this section, we attempt to design the feedback control laws F in order to lead the end-effector to the target position and τ^* in order to control the extra joint motion of redundant manipulators.

Here, we can define the potential function with quadratic form $V_{effector}$ to derive the feedback controller F as follows:

$$V_{effector} = \frac{1}{2}(x^* - x)^T K_1 (x^* - x) + \frac{1}{2}\dot{x}^T K_2 \dot{x}\ , \qquad (7)$$

where x^* denotes the target position of the end-effector, and K_i =diag.$(k_1^i, k_2^i, \cdots, k_m^i)$ under $k_m^i > 0$ $(i = 1, 2)$. When we design the feedback control law F based on the potential function $V_{effector}$ as

$$F = -M_x K_2^{-1}\{K_1(x - x^*) + \dot{x}\} + h_x(q, \dot{q}) + g_x(q)\ , \qquad (8)$$

the time-derivative of $V_{effector}$ yields

$$\dot{V}_{effector} = -||\dot{x}||^2 \leq 0\ , \qquad (9)$$

with the dynamic equation (2). $\dot{V}_{effector}$ is always non-increasing except at the equilibrium point. It follows that the end-effector can reach the target position by the joint torque $\tau_{effector}$ equivalent to the derived control law F given in (8). For a redundant manipulator, however, the joints continue to move although the end-effector arrived at the target position since the designed controller F can not control the extra freedom of joint motion directly. For this problem in redundancy, we apply the null space on the force/torque transformation to control the internal motion.

Here, we define the potential function V_{joint} in order to design the feedback controller τ^* as

$$V_{joint} = \frac{1}{2}\dot{q}^T M \dot{q} + \kappa(t) Q(q)\ , \qquad (10)$$

where $\kappa(t)$ is a positive and non-increasing continuous function, i.e. $\dot{\kappa}(t) < 0$, and $Q(q)$ is a differentiable potential function. The first term on the right side of equation (10) is used in order to dampen the redundant joint motion when the end-effector arrives at the goal, and the second one is used to realize the desired posture of the manipulator q^* corresponding to the minimum/maximum of the potential function $Q(q)$. If we design the feedback control law τ based on the potential function V_{joint} as

$$\tau = -\dot{q} + g(q) - \kappa(t)\frac{\partial Q}{\partial q}\ , \qquad (11)$$

the time-derivative of V_{joint} yields

$$\dot{V}_{joint} = -||\dot{q}||^2 + \dot{\kappa}(t)\ Q(q) \leq 0\ , \qquad (12)$$

with the joint space motion equation (1) and $\dot{\kappa}(t) < 0$ in the actual time scale. Selecting the designed controller $\boldsymbol{\tau}$ (11) as $\boldsymbol{\tau}^*$, we can obtain the joint torque $\boldsymbol{\tau}_{joint}$ to control the internal motion without altering the generating trajectory of the end-effector.

With the total feedback control law $\boldsymbol{\tau}$ given in (6) composed of the designed controller $\boldsymbol{F}$ (8) and $\boldsymbol{\tau}^*$ (11) , the end-effector can be reached the target position and also the desired posture through an optimization procedure of the potential function $Q(\boldsymbol{q})$.

Moreover, substituting (8) into (2), we can derive the following linear damped system:

$$\ddot{\boldsymbol{x}} + \boldsymbol{K}_2^{-1}\dot{\boldsymbol{x}} + \boldsymbol{K}_2^{-1}\boldsymbol{K}_1^{-1}(\boldsymbol{x} - \boldsymbol{x}^*) = 0 \,. \quad (13)$$

Obviously, the system in the operational space (2) is asymptotically stable to the equilibrium point $\boldsymbol{x}^*$ by the designed feedback controller $\boldsymbol{F}$ given in (8). Following the above discussion, we can conclude that it is impossible to regulate the convergence time and the dynamic behavior of the end-effector as hoped [5].

4 APFA with Time Scaling

Generally, the stability and dynamic property of systems has no change in any time scale that is a strictly monotone increasing function with respect to the actual time [8]. This indicates that we can design the feedback control law to converge the original system to the equilibrium point at finite time t_f as long as the asymptotic stabilizer for the system in the new time scale where infinite time corresponds to t_f in the actual time is found.

In this section, we present a detail of the proposed method based on the APFA combined with the time scale transformation.

4.1 Virtual time s and TBG

The relationship between actual time t and virtual time s is given by

$$\frac{ds}{dt} = a(t) \,, \quad (14)$$

where the continuous function $a(t)$, called the time scale function [8], is defined as follows:

$$a(t) = -p\frac{\dot{\xi}}{\xi} \,, \quad (15)$$

where p is a positive constant and $\xi(t)$ is a non-increasing function called the Time Base Generator (TBG) [5][6] generates a bell-shaped velocity profile satisfying $\xi(0) = 1$ and $\xi(t_f) = 0$ with the convergence time t_f. The dynamics of ξ is defined as follows:

$$\dot{\xi} = -\gamma(\xi(1-\xi))^{\beta}, \quad (16)$$

where γ and β is a positive constant under $0 < \beta < 1.0$. The convergence time t_f can be calculated with the gamma function $\Gamma(\,\cdot\,)$ as

$$t_f = \int_0^{t_f} dt = \int_1^0 \frac{d\xi}{\dot{\xi}} = \frac{\Gamma^2(1-\beta)}{\gamma\Gamma(2-2\beta)}. \quad (17)$$

From (14) and (15), the virtual time s can be represented with respect to ξ as follows:

$$s = \int_0^t a(t)\,dt = -p\ln\xi(t) \,. \quad (18)$$

It is obvious that the virtual time s given in (18) never goes backward against the actual time t. We take this virtual time s as a new time scale in time scale transformation.

4.2 Time Scaling of the System

We can rewrite the two dynamic equations in the joint space (1) and in the operational space (2) into the following linear system with the state variable $\boldsymbol{Z} = (\boldsymbol{x}, \boldsymbol{q}, \dot{\boldsymbol{x}}, \dot{\boldsymbol{q}})^T$ as:

$$\frac{d}{dt}\boldsymbol{Z} = \begin{pmatrix} \mathbf{0} & \boldsymbol{I} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}\boldsymbol{Z} + \begin{pmatrix} \mathbf{0} \\ \boldsymbol{I} \end{pmatrix}\begin{pmatrix} \boldsymbol{F}_t \\ \boldsymbol{\tau}_t \end{pmatrix}, \quad (19)$$

where $\mathbf{0} \in \Re^{(m+n)\times(m+n)}$ is the zero matrix and $\boldsymbol{I} \in \Re^{(m+n)\times(m+n)}$ is the unit matrix. Then

$$\boldsymbol{F}_t = \boldsymbol{M}_x^{-1}\left\{\boldsymbol{F} - (\boldsymbol{h}_x(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}_x(\boldsymbol{q}))\right\} , \quad (20)$$

$$\boldsymbol{\tau}_t = \boldsymbol{M}^{-1}\left\{\boldsymbol{\tau} - (\boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}))\right\} \,. \quad (21)$$

The system given in (19) can be rewritten in the virtual time scale s as follows:

$$\frac{d}{ds}\boldsymbol{\Psi} = \begin{pmatrix} \mathbf{0} & \boldsymbol{I} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}\boldsymbol{\Psi} + \begin{pmatrix} \mathbf{0} \\ \boldsymbol{I} \end{pmatrix}\begin{pmatrix} \boldsymbol{F}_s \\ \boldsymbol{\tau}_s \end{pmatrix}, \quad (22)$$

where

$$\boldsymbol{\Psi} = (\psi_1, \psi_2, \psi_3, \psi_4) = (\boldsymbol{x}, \boldsymbol{q}, \frac{\dot{\boldsymbol{x}}}{a(t)}, \frac{\dot{\boldsymbol{q}}}{a(t)})^T, \quad (23)$$

$$\boldsymbol{F}_s = \frac{d}{dt}\left(\frac{1}{a(t)}\right)\dot{\boldsymbol{x}} + \frac{1}{a(t)^2}\boldsymbol{F}_t \,, \quad (24)$$

$$\boldsymbol{\tau}_s = \frac{d}{dt}\left(\frac{1}{a(t)}\right)\dot{\boldsymbol{q}} + \frac{1}{a(t)^2}\boldsymbol{\tau}_t \,. \quad (25)$$

As previously defined in the relationship between actual time and virtual time, stability of the new system given in (22) is the same as the original system in the actual time [8]. Hence, there exists a feedback control law to stabilize the new system asymptotically.

4.3 Design the feedback control law

In this subsection, we design the feedback control law with the APFA to stabilize the new system given in (22) in the virtual time scale.

We can define the potential function with quadratic form $V^{\psi}_{effector}$ for the control of the end-effector in the virtual time scale as follows:

$$V^{\psi}_{effector} = \frac{1}{2}(\psi_1^* - \psi_1)^T K_1(\psi_1^* - \psi_1) + \frac{1}{2}\psi_3{}^T K_2 \psi_3 \,. \tag{26}$$

If we design the feedback controller F_s based on $V^{\psi}_{effector}$ as

$$F_s = -K_2{}^{-1}\{K_1(\psi_1 - \psi_1^*) + \psi_3\} \,, \tag{27}$$

the derivative of $V^{\psi}_{effector}$ with respect to s yields

$$\frac{d}{ds} V^{\psi}_{effector} = -||\psi_3|| \leq 0 \,. \tag{28}$$

By inverse transformation of time scale from the virtual time s to the actual time t for the controller F_s with (22) and (24), the controller F_t in the actual time is derived as

$$F_t = -a^2(t)K_2^{-1}K_1(x - x^*) - \left\{a(t)K_2^{-1} - \frac{\dot{a}(t)}{a(t)}\right\}\dot{x} \,. \tag{29}$$

From (20) and (29), we can obtain the feedback control law F^{ψ} for control of the dynamic behavior of the end-effector as follows:

$$F^{\psi} = M_x F_t + h_x(q, \dot{q}) + g_x(q) \,. \tag{30}$$

The end-effector can be controlled to the target position at the convergence time t_f by means of the joint torque $\tau_{effector}$ equivalent to the feedback control law F^{ψ} given in (30).

On the other hand, we can define the potential function V^{ψ}_{joint} to derive the feedback controller τ_s as

$$V^{\psi}_{joint} = \frac{1}{2}\psi_4{}^T K_3 \psi_4 + \zeta(s)Q_s(\psi_2) \,, \tag{31}$$

where $K_3 = \mathrm{diag.}(k_1^3, k_2^3, \cdots, k_n^3)$ under $k_n^3 \geq 0$, $Q_s(\psi_2)$ is the differentiable potential function in the virtual time scale, and $\zeta(s)$ is a positive non-increasing scalar function. The derivative of V^{ψ}_{joint} with respect to s yields

$$\frac{d}{ds}V^{\psi}_{joint} = \psi_4{}^T\left\{K_3\tau_s + \zeta(s)\frac{\partial Q_s}{\partial \psi_2}\right\} + \frac{d\zeta(s)}{ds}Q_s(\psi_2) \,. \tag{32}$$

If we define the feedback controller τ_s under consideration that the scalar function $\zeta(s)$ is non-increasing in the new time scale as

$$\tau_s = -K_3{}^{-1}\left\{\psi_4 + \zeta(s)\frac{\partial Q_s}{\partial \psi_2}\right\} \,, \tag{33}$$

Equation (32) can be calculated as

$$\frac{d}{ds} V^{\psi}_{joint} = -||\psi_4||^2 + \frac{d\zeta}{ds}(s)Q_s(\psi_2) \leq 0 \,. \tag{34}$$

This indicates that the potential function V^{ψ}_{joint} is stabilized to the equilibrium point by means of the feedback controller τ_s in the virtual time scale.

Here, we define the non-increasing function $\zeta(s)$ as

$$\zeta(s) = \alpha e^{-\frac{2s}{p}} \,. \tag{35}$$

Through inverse time-scale transformation from the virtual time to the actual time for the controller τ_s with (22) and (25), the feedback control law τ_t in the actual time is derived as

$$\tau_t = -\left\{a(t)K_3^{-1} - \frac{\dot{a}(t)}{a(t)}\right\}\dot{q} - \alpha\xi^2(t)a^2(t)K_3^{-1}\frac{\partial Q}{\partial q} \,, \tag{36}$$

where α is a positive constant. From (21) and (36), we can derive the following feedback controller τ^{ψ} as

$$\tau^{\psi} = M\tau_t + h(q, \dot{q}) + g(q) \,. \tag{37}$$

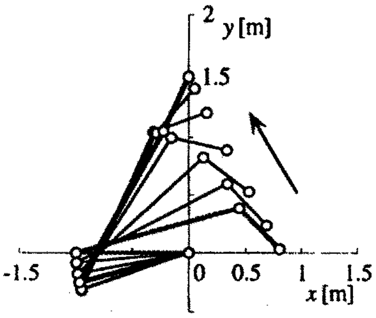
When the joint torque τ^{ψ} (37) is selected as τ^*, we can obtain the joint torque τ_{joint} (5) to control the internal motion of the redundant manipulator in the actual time scale.

The total feedback control law τ (6) composed of the designed controller given in (30) and (37) can lead the end-effector to the target position at the specified time t_f and can also attain the desired posture without altering the configuration of the end-effector.

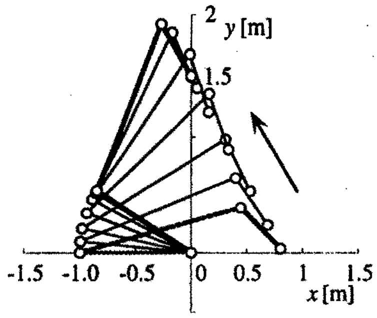
4.4 Dynamic behavior of the end-effector

In this subsection, the dynamic behavior of the end-effector controlled by the feedback controller designed in 4.2 is analyzed. To simplify the discussion, the target position for the end-effector is set at the origin in the operational space. Substituting the feedback control law F^{ψ} given in (30) into the original linear system equation (19), we have the following differential equation as:

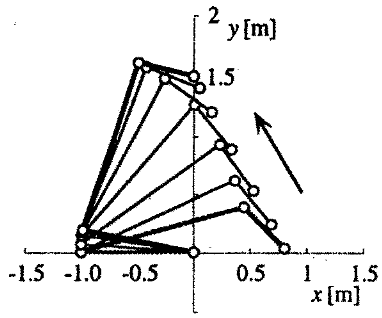
$$\ddot{x} = -p^2\left(\frac{\dot{\xi}}{\xi}\right)^2 K_2^{-1}K_1\, x + \left\{(p-1)\frac{\dot{\xi}}{\xi} + \frac{\ddot{\xi}}{\dot{\xi}}\right\}\dot{x} \,. \tag{38}$$



(a) Case 1



(b) Case 2



(c) Case 3

Figure 1: Changes of the generated end-effector trajectories with different positive functions $Q_i(q)$

Here, we first analyze the behavior of the end-effector on the x coordinate. From (38), the following Euler's equation with respect to x and ξ can be derived as:

$$\xi^2 \frac{d^2x}{d\xi^2} - (p-1)\,\xi\,\frac{dx}{d\xi} + \frac{k_1^1}{k_1^2}p^2 x = 0\,. \quad (39)$$

Since the non-increasing function ξ converges to zero at finite time t_f, the necessary and sufficient condition to converge x, $\dot{x}$ and $\ddot{x}$ to zero at the specified time t_f is given with respect to the discriminant of the characteristic polynomial $D_x = 4\frac{k_1}{k_4} - 1$ as follows:

$$(1)\ if\ D_x \geq 0\ then\ p > 4(1-\beta)\,,$$
$$(2)\ if\ D_x < 0\ then\ p > \frac{4(1-\beta)}{1-\sqrt{-D_x}}\,.$$

The dynamic behavior of the other state variables can be analyzed in the same manner.

It can be proven that the feedback controller F^{ψ} can regulate the dynamic behavior of the end-effector and the convergence time to reach the goal.

5 Computer simulations

The proposed trajectory generation method is applied to a redundant manipulator. Figure 1 shows the simulation results with a three-joint-planar manipulator. The initial posture of the manipulator is $q(0) = (\pi, -\frac{5\pi}{6}, -\frac{\pi}{6})^T$ [rad], and the target position of the end-effector is $x^* = (0.0, 1.5)^T$ [m] with the convergence time $t_f = 5.0$ [s] under $p = 8.0$, $\alpha = 1.0$. The gain matrices K_i $(i = 1,2,3)$ are set at $K_1 = \text{diag.}(0.25, 0.25)$ [N/m], $K_2 = \text{diag.}(1.0, 1.0)$ [Ns/m], and $K_3 = \text{diag.}(1.0,1.0,1.0)$ [Nm/(rad/s)], respectively. We used the Appel method for the manipulator dynamics [13] and the link parameters of the manipulator as shown in Table 1.

Table 1: Link parameters of the manipulator

	link 1	link 2	link 3
length [m]	1.0	1.5	0.5
mass [kg]	0.8	1.2	0.4
center of mass [m]	0.4	0.6	0.25
moment of inertia [kgm^2]	0.06666	0.22500	0.00833

Figure 1 (a) shows the generated trajectory with the potential function $Q(q)$ set at

$$Q_1(q) = 0\,. \quad (40)$$

On the other hand, the joint angle control of the first joint and maximization of the manipulability [11] is considered as a subtask in Fig.1 (b) and (c). In these cases, the potential functions $Q(q)$ are given as

$$Q_2(q) \;=\; \frac{1}{2}(q_1^* - q_1(t))^2\,, \quad (41)$$
$$Q_3(q) \;=\; \sqrt{\det JJ^T}\,, \quad (42)$$

where the target angle of the first joint q_1^* is specified as $q_1^* = \frac{5\pi}{6}$[rad].

It can be seen that the generated trajectories are influenced by the corresponding potential functions $Q(q)$ defined above. In Fig.1 (a), we can observe that the third joint of the manipulator is outstretched while the end-effector reaches the target position.

In contrast, the end-effector reaches the target position without any singular configurations by utilizing the redundancy control of the manipulator corresponding to locall optimization of the potential functions $Q(q)$ in Fig.1 (b) and (c).

Figure 2 shows the time history of the end-effector position x and velocity $\dot{x}$, and the squared sum of the joint angular velocity. It should be noticed that all generated trajectories of the end-effector in Fig.1 completely coincides with the one smooth trajectory

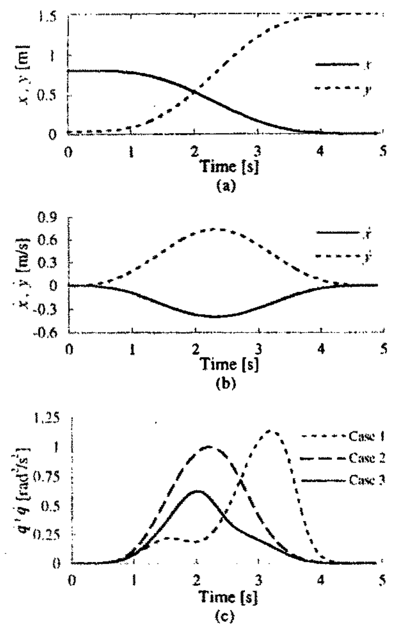


Figure 2: Time histories of the end-effector position, velocity and squared sum of joint velocities

depicted in Fig.2 (a). We can see that the end-effector reached the target position and that the joints of the manipulator do not move any longer after the specified time $t_f = 5.0$[s] in all cases.

6 Conclusions

In this paper, the new trajectory generation method for the dynamic model of redundant manipulators using the concept of the APFA and the time scaling transformation has been presented. We have developed the control strategy for the redundant manipulators that allow achievement of performance with its redundancy. In simulation results with the three joint planar manipulator, the effectiveness of the proposed method was ascertained.

References

[1] L. A. Loeff and A. H. Soni, "An algorithm for computer guidance of a manipulator in between obstacles", Transactions of ASME, Journal of Engineering for Industry, vol.97, no.3, pp.836–842, 1975.

[2] O. Khatib, "Real-time Obstacle Avoidance for Manipulators and Mobile robots", int.J.Robotics Res., Vol.5-1, pp.396–404, Spring (1986).

[3] C. I. Connolly, J. B. Burns and R. Weiss, "Path planning using laplace's equation", in Proceedings of IEEE International Conference on Robotics and Automation, pp.2102–2106, 1990.

[4] H. Hashimoto, Y. Kunii, H. Harashima, V. I. Utkin, S. A. Krasnova, and I. M. Kaliko,"Sliding mode control and potential fields in obstacle avoidance", in Proceedings of European Control Conference, pp.859–862, 1993.

[5] T. Tsuji, P. G. Morasso and M. Kaneko, "Trajectory Generation for Manipulators Based on Artificial Potential Field Approach with Adjustable Temporal Behavior", in Proceedings of IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, pp.438–443, 1996.

[6] T. Tsuji, P. Morasso and M. Kaneko, "Feedback control of nonholonomic mobile robots using time base generator", in Proceedings of IEEE Int. Conf. on Robotics and Automation, pp.1385–1390, 1995.

[7] J. M. Hollerbach, "Dynamic Scaling of Manipulator Trajectories", Transactions of ASME, Vol.106, pp.102–106, 1984.

[8] M. Sampei and K. Furuta, "On time scaling for nonlinear systems: Application to linearization", IEEE transactions on Automatic Control, vol.AC-31, no.5, pp.459–462, 1986.

[9] Y. Tanaka, T. Tsuji, M. Kaneko, "Dynamic control of ,mobile robots via artificial potential field approach with time scaling", '98 IEEE International Conference on Robotics and Automation, 1998 (Submitting).

[10] A. Liegeois, "Automatic supervisory control of the configuration and behavior of multibody mechanisms", Transactions of SMC, Vol.7, No.12, pp.868–871, 1977

[11] T. Yoshikawa, "Analysis and Control of Robot Manipulators with Redundancy", Robotics Research International Symposium, Brady and Paul eds., MIT Press, pp.735–747, 1984.

[12] O.khatib, "Motion/Force Redundancy of Manipulators, Proc. of U.S.A Symposium on Flexible Automation", Vol.1, pp.337–342, 1990.

[13] V. Potkonjak and M. Vukobratovic, "Two new methods for computer forming of dynamic equation of active mechanisms", Mechanism and Machine Theory, Vol.14, No.3, pp.189–200, 1979.