Trajectory Generation for Manipulators Based on Artificial Potential Field Approach with Adjustable Temporal Behavior

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Abstract

This paper presents a new trajectory generation method of the artificial potential field approach to a real-time motion planning problem. In the artificial potential field approach, the goal is represented by an attractive artificial potential and the obstacles are represented by repulsive ones, so that the robot reaches the goal without colliding with obstacles by using a gradient vector of the potential field. Although this approach is simple and computationally much less expensive than other methods based on the global information on the task space, few attention have been paid to temporal behavior of the generated trajectories such as a movement time from the initial position to the goal and the velocity profile of the end-effector motion. In the present paper, we argue that temporal behavior of the generated arm trajectory should be taken into account within the framework of the artificial potential field approach and introduce a time base generator that determines a dynamic behavior of the arm trajectory as a part of a feedback controller. By synchronizing a time course of the potential function used as an artificial potential field with the time base generator, the temporal behavior of the generated arm trajectory can be adjusted through the time base generator without any change of the potential function to be used.

1 Introduction

In the artificial potential approach [1] – [7] used for a trajectory generation problem of a mobile robot or a manipulator, the goal is represented by an attractive artificial potential and the obstacles are represented by repulsive ones, so that the trajectory from the initial position to the goal can be generated by using a gradient vector of the potential field. This approach is suitable for a real-time motion planning of robots since the algorithm is simple and computationally much less expensive than other methods based on the global information on the task space. However, at least two important disadvantages should be pointed out.

The first one is the local minima problem. At the point where the attractive potential to the goal and the repulsive one from the obstacles are equal, the gradient vector of the potential field becomes zero and the robot falls into a deadlock. Many methods have been proposed to overcome this problem. For example, Okutomi and Mori [3] used an ellipse-shape potential field and showed that the possibility falling into local minima could be reduced by devising the potential function. Connolly, Burns and Weiss [4] proposed a method using the Laplace's differential equation based on the idea that the local minima problem is completely solved when the potential function not including any local minima can be defined. Also, some potential functions on the basis of this idea have been proposed [5], [6].

The other disadvantage is that a temporal behavior of the generated trajectories such as a movement time from the initial position to the goal and a velocity profile of the end-effector motion cannot be regulated by the artificial potential approach. For example, even if the use of the potential function not including local minima can assure that the manipulator always reaches the goal, it is difficult to estimate the movement time required for reaching beforehand. Also the velocity profile of the generated trajectory cannot be adjusted as we want since it is determined by the shape of the potential field. Although one of the most important features of the artificial potential field approach is a real-time applicability, it is difficult to use the generated trajectory for the control of robots in real time because of this disadvantage. There has been not enough consideration about the temporal behavior of the generated trajectories since the deadlock-free path planning was the most active research topic about the artificial potential field approach.

Recently, Hashimoto et al. [7] proposed the method using the electrostatic potential field and the sliding mode control. The method can generate a continuous trajectory not including local minima and has an excellent feature that it assures to reach the goal in a finite time. Also, Morasso, Sanguineti and Tsuji [8], [9] proposed the two-dimensional trajectory generation model for reaching movements of human arm. In this method, the hand trajectory is generated by synchronizing a translation velocity and a rotational angular velocity of the hand with a signal generated by a Time Base Generator (TBG). The TBG is a

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0-7803-3213-X/96/ $ 5.00 © 1996 IEEE

time series generator that the generated signal has a finite duration and shows a bell-shaped velocity profile. Thus, the movement time and the velocity profile of the hand trajectory can be regulated indirectly by changing a dynamic property of the TBG.

This paper proposes a new trajectory generation method by introducing the TBG into the artificial potential field approach for multi-joint manipulators. The method can regulate the movement time of the manipulator from the initial position to the goal and the velocity profile of the end-effector motion. Various potential functions developed in the previous researches can be used in the proposed method since the temporal behavior of the generated trajectory is regulated through the TBG rather than the potential function.

2 Trajectory Generation Using Time Base Generator

2.1 Artificial Potential Field Approach on the Basis of the Steepest Descend Method

Let us consider a trajectory generation problem to move the end-effector of the m-joint manipulator shown in Fig.1 from the initial position x_0 to the target position x^*. Note that only the kinematics of the manipulator is considered in this paper to make the problem simpler. Now, the relationship between the joint angle vector $\theta \in \mathrm{R}^m$ and the end-effector position vector $x \in \mathrm{R}^l$ is given by

$$x = f(\theta), \tag{1}$$

where $f(\cdot)$ is a nonlinear function representing the kinematics of the manipulator.

Then, a differentiable potential function $V(x)$ is defined in the task space. At the target position x^*, the potential function becomes zero, $V(x^*) = 0$, and at other end-effector positions in the task space, $V(x)$ is larger than 0.

Motion planning can be performed on the basis of the artificial potential field defined above in two different ways: the resolved motion rate control approach

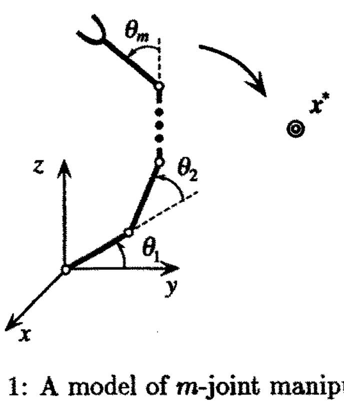

Figure 1: A model of m-joint manipulator

[10] and the force control approach [2], [11]. In the resolved motion rate control approach, the end-effector velocity vector in the direction of the negative gradient field is computed as $\dot{x} = -\frac{\partial V}{\partial x}$. Then, the end-effector velocity vector $\dot{x}$ is transformed into the joint velocity vector $\dot{\theta}$ through kinematic inversion. On the other hand, the antigradient field $-\frac{\partial V}{\partial x}$ is used as an artificial end-effector force field acting on the end-effector in the force control approach. Then, the resulting force vector is converted to the joint torque vector τ using the transposed Jacobian matrix and dynamic evolution of the manipulator results the end-effector motion towards the target position.

The first approach requires to solve the inverse kinematics problem, which is computationally expensive especially for redundant manipulators. On the other hand, the second approach is simple and effective, although the dynamic effects must be evaluated.

In this paper, on the basis of the kinematics of the manipulator, the joint velocity vector $\dot{\theta}$ is determined through an optimization procedure of the potential function $V(x)$. Using the negative gradient vector of $V(x)$ in the direction of the steepest descent, we define the joint velocity vector as follows:

$$\dot{\theta} = -\eta \left(\frac{\partial V}{\partial \theta}\right)^T = -\eta \left(\frac{\partial V}{\partial x} J\right)^T, \tag{2}$$

where η is a positive constant and $J \in \mathrm{R}^{l \times m}$ is the Jacobian matrix.

The time derivative of the potential function V can be calculated as

$$\begin{aligned} \dot{V} &= \frac{\partial V}{\partial x}\frac{\partial x}{\partial \theta}\dot{\theta} \\ &= -\eta \left(\frac{\partial V}{\partial x} J\right)\left(\frac{\partial V}{\partial x} J\right)^T \\ &= -\eta \parallel \frac{\partial V}{\partial x} J \parallel^2 \end{aligned} \tag{3}$$

Assuming that the manipulator is not in any singular configurations and the potential function $V(x)$ is chosen in such a way that the gradient vector $\frac{\partial V}{\partial x}$ is not zero at any end-effector positions in the task space except for the target position, we can assure that the system is asymptotically stable.

Although the amplitude of the joint velocity can be changed using the parameter η of (2), a criterion to determine the parameter η is not given. It is impossible to specify the time required to reach the target position precisely and impossible to regulate the velocity profile of the end-effector to a desirable one.

2.2 Use of Time Base Generator

At first, the scalar signal $\xi(t)$ is defined as a first differentiable and monotonically non-increasing function satisfying $\xi(0) = 1$ and $\xi(t_f) = 0$, where t_f represents the convergence time. In this paper, we consider the regulation of the temporal behavior of the generated trajectory by synchronizing the time course of the potential function V with the scalar signal $\xi(t)$ and a mechanism generating $\xi(t)$ is called a Time Base Generator (TBG) [8].

Using this $\xi(t)$, (2) is modified as follows:

$$\dot{\theta} = \frac{pV\dot{\xi}}{\xi}\frac{\left(\frac{\partial V}{\partial x}J\right)^T}{\|\frac{\partial V}{\partial x}J\|^2}, \tag{4}$$

where p is a positive constant. This means that the parameter η in the artificial potential field approach (2) is selected as follows:

$$\eta = -\frac{pV\dot{\xi}}{\xi\|\frac{\partial V}{\partial x}J\|^2}. \tag{5}$$

Then, the time derivative of the potential function V can be calculated as

$$\dot{V} = pV\frac{\dot{\xi}}{\xi}. \tag{6}$$

Since $p > 0$, $V > 0$, $\xi > 0$ and $\dot{\xi} < 0$ at any t except for $t = t_f$, we have $\dot{V} < 0$, that is, asymptotic stability of the system is assured.

Equation (6) can be transformed as

$$\frac{dV}{d\xi} = p\frac{V}{\xi}. \tag{7}$$

The above differential equation can be solved for the variable ξ as follows:

$$V = V_0\xi^p, \tag{8}$$

where $V_0 = V(x_0)$ is the initial value of V. Thus we can see that the potential function V is proportional to the pth power of ξ. Since ξ reaches zero at t_f, the end-effector of the manipulator must arrive at the target position x^* at t_f where the potential function also becomes zero $V = 0$.

In the limit when ξ approaches zero, the first derivative of the potential function is given as

$$\lim_{\xi\to 0}\dot{V} = -\xi^{p+\beta-1}. \tag{9}$$

We can see that $\dot{V}$ converges to zero at t_f if the parameter p is selected as $p \geq 1-\beta$. Also the second derivative of the potential function can be computed as

$$\lim_{\xi\to 0}\ddot{V} = -\xi^{2\beta+p-2}(p+\beta-1), \tag{10}$$

and $p \geq 2(1-\beta)$ is the condition that it converges to zero.

2.3 An Example of TBG

As an example of the TBG, the terminal attractor is used in this paper. The terminal attractor was introduced by Zak [12] into a non-linear neural network model. It has been shown that the system with the terminal attractor always converges to the equilibrium point in a finite time at which the Lipschitz condition is violated.

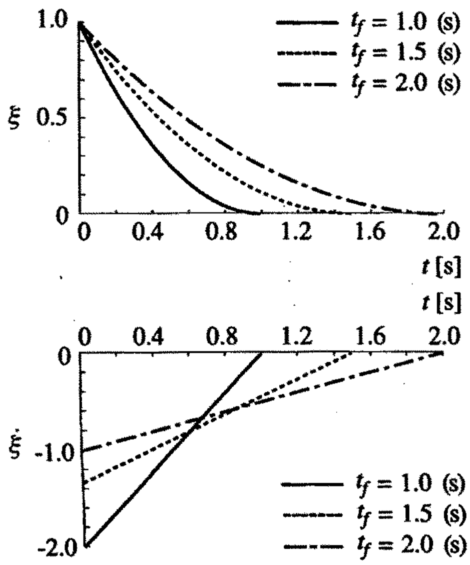

Figure 2: Change of the ξ behavior depending on the convergence time t_f with the constant power parameter $\beta = 0.5$

Here, the following nonlinear dynamics is considered as the TBG with the terminal attractor:

$$\dot{\xi} = -\alpha\xi^{\beta}, \tag{11}$$

where $\alpha > 0$ and $0 < \beta < 1$ are constants and $\xi(0) = 1.0$ from the definition. Then, we have

$$\left|\frac{d\dot{\xi}}{d\xi}\right| = \alpha\beta\xi^{\beta-1}, \tag{12}$$

so that the Lipschitz condition is violated at the equilibrium point $\xi \to 0$ [11]. The convergence time t_f of $\xi(t)$ can be calculated as follows:

$$t_f = \int_0^{t_f} dt = \int_1^0 \frac{d\xi}{\dot{\xi}} = \frac{1}{\alpha(1-\beta)}. \tag{13}$$

Consequently, we can see that the equilibrium point $\xi = 0$ is a terminal attractor because the convergence time is always given as the finite value. Using (13), the parameter α in (11) is determined as

$$\alpha = \frac{1}{t_f(1-\beta)}, \tag{14}$$

so that the convergence time t_f can be specified precisely.

Figures 2 and 3 indicate the temporal behavior of the scalar signal ξ generated by the TBG when the parameters t_f and β are varied. In Fig. 2, the time

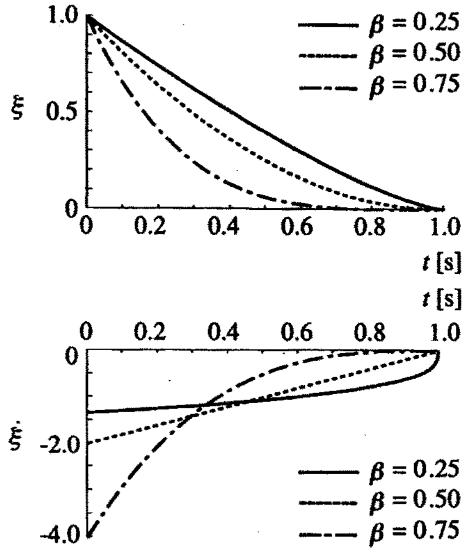

Figure 3: Change of the ξ behavior depending on the power parameter β with the constant convergence time $t_f = 1.0$[s]

histories of ξ and $\dot{\xi}$ with different convergence times $t_f = 1.0$, 1.5 and 2.0(s) are shown, where $\beta = 0.5$. All trajectories converge to the equilibrium point at the specified time t_f. Also, Fig. 3 shows the time histories of ξ and $\dot{\xi}$ with different power parameters $\beta = 0.25$, 0.5 and 0.75, where the convergence time is fixed at $t_f = 1.0$(s). The time history of $\dot{\xi}(t)$ can be regulated through the power parameter β with the same convergence time.

Consequently, using the TBG with two parameters t_f and β presented here, we can generate various time-varying signals $\xi(t)$. In the following section, the method proposed in this paper is applied for a redundant manipulator and an obstacle avoidance problem.

3 Simulation Experiments

3.1 Application to Redundant Manipulator

The proposed trajectory generation method is applied to a redundant manipulator. First, in order to utilize arm redundancy, (4) is modified as

$$\dot{\theta} = \frac{pV\dot{\xi}}{\xi}\frac{\left(\frac{\partial V}{\partial x}J\right)^T}{\| \frac{\partial V}{\partial x}J \|^2} - \gamma(I - J^+J)\frac{\partial V_s}{\partial \theta}, \tag{15}$$

where γ is a scalar function that satisfies $\gamma(t) \geq 0$ in $t = [0, t_f]$ and $\gamma(t_f) = 0$. Also V_s is a differentiable potential function, which is locally minimized using redundant joint degrees of freedom. The first term in the right side of (15) is the joint velocity vector for the control of the end-effector trajectory, and the second term works in order to reduce the potential function V_s in such a way that the end-effector trajectory is not affected.

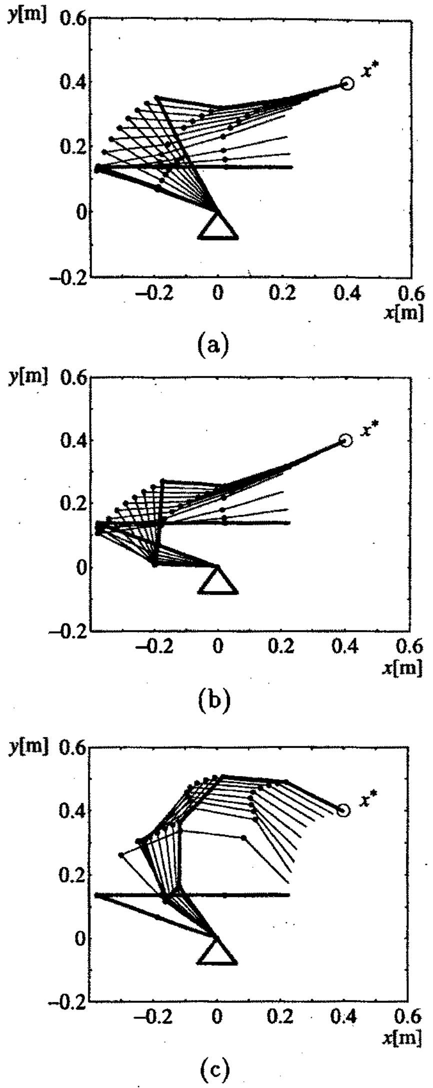

Figure 4: Generated trajectories for the five-joint planar manipulator

Figure 4 shows examples of the generated trajectories using the five-joint plane manipulator, where length of each link is 0.2 m. The initial posture of the manipulator is $\theta = [\frac{8\pi}{9}, 0, -\frac{8\pi}{9}, 0, 0]^T$(rad), and the target position of the end-effector is $x^* = [0.4, 0.4]$(m). The potential function $V(x)$ used here is simply defined as

$$V(x) = \frac{1}{2}dx^T dx, \tag{16}$$

where $dx = x^* - x$. Also the parameters $\beta = 0.5$, $p = 1$ and the convergence time $t_f = 1.0$(s) are used.

Figure 4 (a) shows the generated trajectory using (4), which means that arm redundancy is not utilized. Time history of the end-effector position is shown in Fig. 5. It can be seen that the end-effector reaches the target position at the prespecified time $t_f = 1.0$(s).

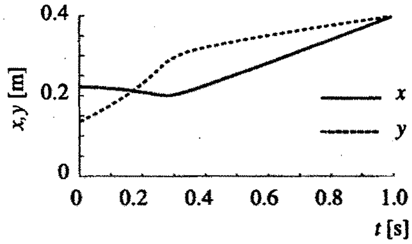

Figure 5: Time history of the end-effector motion

On the other hand, (15) is used in Fig. 4 (b) and (c) with $\gamma(t) = 200(1.0 - t)$. Maximization of the manipulability [13] and position control of the second joint of the manipulator are considered as a subtask in Fig. 4 (b) and (c), respectively. In this case, the potential functions V_s used in (15) are given as

$$V_{s1} = -\sqrt{\det JJ^T} \tag{17}$$

$$V_{s2} = \frac{1}{2} dx_v^T dx_v, \tag{18}$$

where $dx_v = x_v^* - x$ is the error vector between the position of the second joint and its target position. In Fig. 4 (c), the target position is specified as $x_v^* = [-0.3, -0.1]^T$(m).

It can be seen from the figures that the generated arm trajectories are influenced by the corresponding subtask defined above. Note that the end-effector trajectory in each figure reaches the target position at the specified time $t_f = 1.0$(s) and the time history of $V(x)$ is exactly the same as the one of Fig. 4 (a).

3.2 Trajectory Generation in the Task Space Including Obstacles

Next, the proposed method is applied to the obstacle avoidance problem. Using the harmonic potential function [6], the artificial potential field for the task space including obstacles is defined as

$$V(x) = \lambda_g \log(\| dx \|) - \sum_{i=1}^{n} \lambda_{oi} \log(\| dx_{oi} \|), \tag{19}$$

where the obstacles are represented as n points x_{oi} and the displacement between x and x_{oi} is defined as $dx_{oi} = x_{oi} - x$. Also λ_g and λ_{oi} are positive constants, which satisfy $\lambda_g \geq \sum_{i=1}^{n} \lambda_{oi}$, and $\log(\cdot)$ is the natural logarithm.

The first term of the right side (19) represents an attractive potential to the goal and the second term represents a repulsive one from the obstacles. It can be easily seen that (19) satisfies the Laplace's equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0, \tag{20}$$

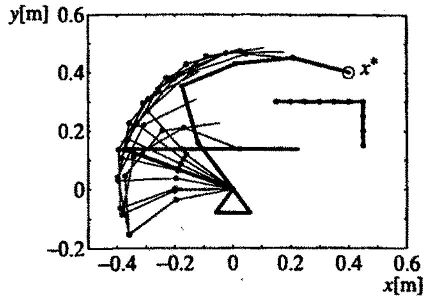

Figure 6: Real-time obstacle avoidance for the five-joint planar manipulator generated by the proposed method

so that there is no local minimum in the task space. Then we can have

$$\frac{\partial V}{\partial x} = -\frac{\lambda_g}{\| dx \|^2} dx^T + \sum_{i=1}^{n} \frac{\lambda_{oi}}{\| dx_{oi} \|^2} dx_{oi}^T. \tag{21}$$

Using the potential function (19), it is possible to reach the goal without falling into the deadlock [4] – [6]. However, because this potential function approaches negative infinity $V(x^*) \to -\infty$ at the target position x^*, (4) or (15) cannot be used directly. Therefore, (15) is modified as

$$\dot{\theta} = \frac{p\dot{\xi}}{\xi} \frac{\left(\frac{\partial V}{\partial x} J\right)^T}{\| \frac{\partial V}{\partial x} J \|^2} - \gamma(I - J^+ J)\frac{\partial V_s}{\partial \theta}. \tag{22}$$

Then the time derivative of the potential function can be derived as

$$\dot{V} = p\frac{\dot{\xi}}{\xi}. \tag{23}$$

Since $\xi > 0$, $p > 0$ and $\dot{\xi} < 0$ at any t except for $t = t_f$, we have $\dot{V} < 0$.

Solving (23) for V, we have

$$V = V_0 + p \log \xi. \tag{24}$$

When $\xi \to 0$, $V \to -\infty$, so that the end-effector of the manipulator reaches the target position x^*.

Figure 6 shows an example of the generated trajectory using this potential function. The same manipulator model as the one in Fig. 4 is used, and the potential function V_s for the subtask of (22) is defined as

$$V_3 = \sum_{j=1}^{4} \Phi_j = -\sum_{j=1}^{4} \sum_{i=1}^{n} \lambda_{oi} \log(\| dx_{oi}^{vj} \|), \tag{25}$$

where $dx_{oi}^{vj} = x_{oi} - x_{vj}$ is the displacement between the position of the j-th arm point set on the manipulator x_{vj} and the i-th point obstacle x_{oi}. Four arm points are set on each joint except for the first joint and ten

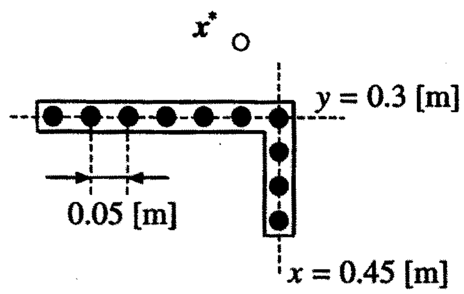

Figure 7: Goal and obstacle

point obstacles are used as shown in Fig. 7. The target position of the end-effector is $x^* = [0.4, 0.4]$(m). The potential function V_s for the subtask is effective for avoiding a collision between the obstacle and the whole body of the manipulator. Note that parameter $\beta = 0.1$, $\lambda_g = 3.75$, $\lambda_{oi} = 0.2$ $(i = 1, 2, \cdots, 10)$ are used and other experimental conditions are the same as Fig. 4. From Fig. 6, we can see that the end-effector reaches the goal avoiding obstacles.

Obviously, the method presented in this section may fall into a deadlock for some environments since the motion planning is done in the task space. Although the potential function without any local minima is used for the end-effector motion, other part of the manipulator may collide with the objects. We introduced this method In order to give an example to show how the TBG can be utilized for the obstacle avoidance problem. If the deadlock problem must be solved, we can easily apply our method in the configuration space approach using the potential field defined in the joint configuration space of the manipulator.

4 Conclusion

In this paper, the trajectory generation method in the basis of the artificial potential field approach has been proposed. The method can regulate the temporal behavior such as the movement time and the velocity profile of the generated trajectory by synchronization of the time course of the potential function used as an artificial potential field with the TBG. Then the method was applied to the trajectory generation problem of the redundant manipulator for the task space including the obstacles. It was shown that the method matches other control strategies such as utilization of the arm redundancy and deadlock-free potential fields. The method proposed here is also effective for coordination of multiple robots and path planning of mobile robots [14] since it can regulate the temporal trajectory as well as the spatial trajectory of the robots.

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