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Dynamic Simulation of Multi-Arm Robots Using Appel's Method

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Abstract— **A method for the dynamic simulation of multi-arm robots by taking the mechanism of various contact types into consideration is proposed. The system of multi-arm robots grasping a common object forms a multiple closed-chain mechanisms. Each arm and the object can be modeled to be an open-chain with kinematic constraints on its end-effector motion. The dynamics of each arm is expressed using the Appel's method, where the end-effector's constraints of each arm are derived from the object motion. As a result, a parallel computation of joint acceleration for each arm can be performed , and various contact types between the end-effectors and the object can be expressed by constraint equations. Numerical examples are carried out to evaluate the validity of the proposed method.**

I. Introduction

The dynamic computer simulation of the multi-arm robots have been studied by several researchers. For example, Anderson [1] presented a method for the forward dynamics of two cooperating manipulators by regarding the interactive forces as the external forces exerted on the end-effectors. Oh and Orin [2], Rodriguez [3] and Murphy et al. [4] developed the methods of dynamic computer simulation for an arbitrary number of robot arms grasping a common object. It is noted that an unified approach was formulated by Oh and Orin [2], such that a single set of equations may describe dynamics of both multi-arm robots and legged vehicles. However, all of the previous works have concentrated on the multi-arm robots with only the rigid contact between the end-effectors and the object.

The present paper analyzes the dynamic equations of motion for multi-arm robots grasping a common object, by taking the mechanism of contact-type into consideration, such that the method presented here can be applied for various contact-types between the end-effectors and the object. Also, this method allows each arm of the multi-arm robots to be simulated in a parallel way using the Appel's method.

When the multi-arm robot grasping a common object, multiple closed-chain mechanism will be formed. Each arm and the object can be considered as an open-chain with kinematic constraints on its end-effector motion. Using the forward dynamics of each open-chain, the motion equations of the object, the kinematic relationships between the object and the end-effectors and the kinematic relationships between the end-effectors and the joints, the joint accelerations can be calculated. The forward dynamics of each open-chain can be derived separately using the Appel's method, where the acceleration constraints of each end-effector are obtained from the object motion. Therefore, each arm can be simulated simultaneously. At the same time, the various contact-types can be expressed in the kinematic relationships between the object and the end-effectors.

First of all, the application of the Appel's method for dynamic analysis of a single-arm robot with kinematic constraints is reviewed, followed by the derivation of the kinematic relationships of the multi-arm robots. Then, the forward dynamics of multi-arm robots grasping a common object is derived. Finally, numerical analysis are carried out and the validity of this method is shown.

II. Dynamic Model of Single–Arm Robot : Unconstrained Case

In this section, the single-arm dynamic model for the unconstrained end-effector based on Appel's equations will be reviewed. This model was developed in its final form by Potkonjak and Vukobratovich [5].

Consider a mechanism of the open-chain type, formed by n rigid bodies of arbitrary form, without branching, and mechanism segments are interconnected by the rotational or translational joint, such as shown in Fig.1. The kinematic variables, such as angular velocity $\tilde{\omega}_i$, angular acceleration $\tilde{\varepsilon}_i$, and linear acceleration $\tilde{a}_i$ are determined with respect to the local coordinate system (the body-fixed coordinate system) of the i-th link. The relations determining these kinematic quantities are derived in the same way as in the Newton-Euler's method

$$\bar{\omega}_i = A_{i,i-1}\bar{\omega}_{i-1} + \dot{q}_i(1-s_i)\bar{e}_i \,, \qquad (1)$$

$$\bar{\varepsilon}_i = A_{i,i-1}\bar{\varepsilon}_{i-1} + [\ddot{q}_i\bar{e}_i + \dot{q}_i(\bar{\omega}_i\times\bar{e}_i)]\,(1-s_i) \,, \qquad (2)$$

$$\bar{a}_i = A_{i,i-1}[\bar{a}_{i-1} - \bar{a}_{i-1}\times\bar{r}_{i-1,i} - \bar{\omega}_{i-1}\times(\bar{\omega}_{i-1}\times\bar{r}_{i-1,i})] + \bar{\varepsilon}_i\times\bar{r}^l_{ii} + \bar{\omega}_i\times(\bar{\omega}_i\times\bar{r}^l_{ii}) + [\ddot{q}_i\bar{e}_i + 2\dot{q}_i\,(\bar{\omega}_i\times\bar{e}_i)]s_i \,, \qquad (3)$$

$$\bar{r}^l_{ii} = \bar{r}_{ii} + q_i\bar{e}_i s_i \,, \qquad (4)$$

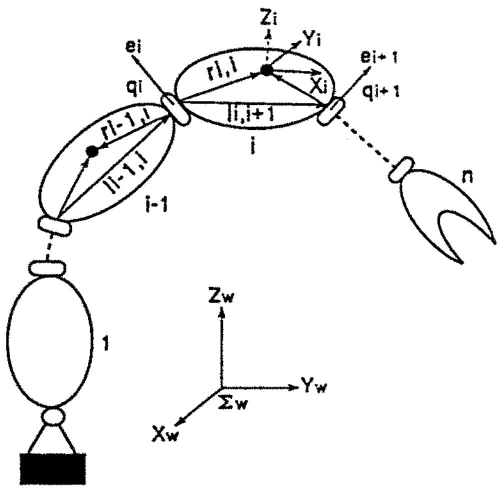

Fig.1 Single-arm robot: unconstarined case

where A_{ij} is the transition matrix from the j-th body-fixed coordinate system to the i-th body-fixed coordinate system and s_i is the joint type ($s_i = 0$ if the joint is rotational, and $s_i = 1$ if the joint is translational). $\bar{e}_i$ denotes the unit vector of the joint axis respect to the body-fixed coordinate system, $\bar{r}_{i-1,i}$ denotes the vector from the i-th joint to the center of mass of the (i-1)-th link respect to the body-fixed coordinate system and q_i denotes the generalized coordinate of the i-th joint. "×" denotes the vector cross product.

By introducing the matrices Ω, Φ, β, Θ and the generalized coordinate vector $q = [q_1 \; q_2 \; \cdots \; q_n]^T$, we can write the linear acceleration and the angular acceleration in the form :

$$\bar{a}_i = \beta\,\ddot{q} + \Theta \,, \qquad (5)$$

$$\bar{\varepsilon}_i = \Omega\,\ddot{q} + \Phi \,, \qquad (6)$$

where $\beta = [\beta^i_1 \; \beta^i_2 \; \cdots \; \beta^i_i \; 0 \; 0 \; \cdots \; 0]$, $\Theta = [\delta^i]$, $\Omega = [\Omega^i_1 \; \Omega^i_2 \; \cdots \; \Omega^i_i \; 0 \; 0 \; \cdots \; 0]$ and $\Phi = [\gamma^i]$. Ω^i_j, β^i_j, δ^i, and γ^i are derived from the recursive expressions (1) - (4). Thus, for the i-th iteration:

$$\Omega^i_i = \bar{e}_i(1-s_i) \,, \qquad (7)$$

$$\beta^i_i = \Omega^i_i\times\bar{r}_{ii} + \bar{e}_i s_i \,, \qquad (8)$$

$$\Omega^i_j = A_{i,i-1}\,\Omega^{i-1}_j \;\; ; \;\; j = (1,2,\cdots(i-1)) \,, \qquad (9)$$

$$\gamma^i = A_{i,i-1}\,\gamma^{i-1} + \dot{q}_i(\bar{\omega}_i\times\bar{e}_i)\,(1-s_i) \,, \qquad (10)$$

$$\beta^i_j = A_{i,i-1}\beta^{i-1}_j + A_{i,i-1}\Omega^{i-1}_j\times\bar{r}_{i-1,i} + \Omega^i_j\times\bar{r}_{ii} \,, \qquad (11)$$

$$\delta^i = A_{i,i-1}\delta^{i-1} + A_{i,i-1}\,(\gamma^{i-1}\times\bar{r}_{i-1,i}) + \gamma^i\times\bar{r}_{ii} + h_i \qquad (12)$$

$$h_i = -A_{i,i-1}\bar{\omega}_{i-1}\times(\bar{\omega}_{i-1}\times\bar{r}_{i-1,i}) + \bar{\omega}_i\times(\bar{\omega}_i\times\bar{r}^l_{ii}) + 2\dot{q}_i\,(\bar{\omega}_i\times\bar{e}_i)\,s_i \,. \qquad (13)$$

Now, using (5) and (6), the Gibbs-Appel's "acceleration energy" function, S, for the single-arm robot will have the form:

$$S = \tfrac{1}{2}\,\ddot{q}^T W\,\ddot{q} + V\,\ddot{q} + D \,, \qquad (14)$$

where $W = \sum_{i=1}^{n} W_i$, $V = \sum_{i=1}^{n} V_i$ and $D = \sum_{i=1}^{n} D_i$. W_i, V_i, and D_i are given by

$$W_i = m_i\Omega^T\Omega + \beta^T\bar{I}_i\beta \,, \qquad (15)$$

$$V_i = m_i\Theta^T\Omega + \Phi^T\bar{I}_i\beta - \bar{u}^T\beta \,, \qquad (16)$$

$$D_i = \tfrac{1}{2}\,m_i\Theta^T\Theta + \tfrac{1}{2}\,\Phi^T\Phi - \bar{u}^T\Phi \,, \qquad (17)$$

$$\bar{u} = (\bar{I}_i\cdot\bar{\omega}_i)\times\bar{\omega}_i \,, \qquad (18)$$

where $\bar{I}_i$ is the inertia tensor of the i-th link respect to the body-fixed coordinate system, m_i is the mass of the i-th link and " · " denotes the vector dot product.

On the other hand, the Appel's equation can be written in the matrix form

$$\frac{\partial S}{\partial \ddot{q}} = Q \,, \qquad (19)$$

where Q is the vector of generalized forces. Substituting (14) into (19), we can obtain the dynamic equation of the unconstrained single-arm robot

$$W\,\ddot{q} = Q - V^T \,. \qquad (20)$$

The vector Q has the form

$$Q = P + Y \,, \qquad (21)$$

where P is the joint torque vector. $Y = [y_1 \; y_2 \; \cdots \; y_n]^T$ is the gravitational torque vector which calculated independent of P, and given by

$$y_i = (e_i\cdot g\sum_{k=0}^{n-1} m_{i+k})s_i + (1-s_i)\sum_{k=0}^{n-1}|m_{i+k}\,g \,,\; e_i \,,\; r^i_k| \qquad (22)$$

$$r^i_k = \sum_{p=0}^{k} r^l_{i+p,i+p} - \sum_{p=0}^{k-1} r_{i+p,i+p+1} \,, \qquad (23)$$

where "| |" denotes the vector box product and $g\in R^3$ is the gravity acceleration vector. It should be noted that the generalized forces are calculated recursively in the algorithm, and this calculation can be performed in the world coordinate system or in the body-fixed coordinated system.

III. Representation for the Constrained Case

Now, we show how the results obtained for the unconstrained case can be used for solving the dynamics of the single-arm robot, where some or all of the end-effector directions are constrained by the environment. This method was proposed by Masuda et al. [6].

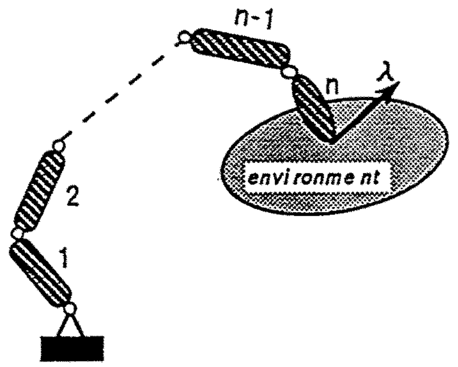

Fig.2 Single-arm robot with kinematic constarints on its end-effector

Consider a single-arm robot constrained by the environment shown in Fig.2. In this case, there are interaction forces/moments between the end-effector and the environment according to the constrained directions.

Let us define $\lambda \in R^{l_c}$ is the vector of the forces/moments exerted on the constrained end-effector, and l_c is the number of the end-effector directions constrained by the environment. We treat these forces/moments as the external forces/moments acting to the arm, such that the dynamic equation of the robot arm becomes

$$W\ddot{q} = P + Y - V^T + J_c^T\lambda\,, \qquad (24)$$

where $J_c \in R^{l_c \times n}$ is the Jacobian matrix corresponding to the constrained directions of the end-effector.

On the other hand, we have the kinematic relationships for the constrained directions such as given by

$$\dot{X}_c = J_c\dot{q}\,, \qquad (25)$$

where $\dot{X}_c$ is the constrained end-effector velocity vector. By differentiation we can find

$$\ddot{X}_c = J_c\ddot{q} + \dot{J}_c\dot{q}\,. \qquad (26)$$

So, we can rewrite (24) and (26) in the compact form

$$\begin{bmatrix} W & -J_c^T \\ -J_c & 0 \end{bmatrix}\begin{bmatrix} \ddot{q} \\ \lambda \end{bmatrix} = \begin{bmatrix} P+Y-V^T \\ \dot{J}_c\dot{q}-\ddot{X}_c \end{bmatrix}, \qquad (27)$$

where $\ddot{X}_c$ can be computed from the constraint condition. For example, if the end-effector cannot move in any direction, then

$$\ddot{X}_c = 0\,. \qquad (28)$$

Equation (27) expresses the dynamic equation of the single-arm robot constrained by the environment. The Jacobian matrix J_c and $\dot{J}_c\dot{q}$ can be computed using the recursive expressions which are similar as in (7) - (13).

The manipulator mass matrix W is a positive definite matrix and invertible, such that the coefficient matrix of (27) is not singular. Therefore, the joint acceleration vector $\ddot{q}$ and the force/moment vector λ can be solved by

$$\begin{bmatrix} \ddot{q} \\ \lambda \end{bmatrix} = \begin{bmatrix} W & -J_c^T \\ -J_c & 0 \end{bmatrix}^{-1}\begin{bmatrix} P+Y-V^T \\ \dot{J}_c\dot{q}-\ddot{X}_c \end{bmatrix}. \qquad (29)$$

IV. Dynamic Simulation for Multi–Arm Robots

The multi-arm robot grasping a common object to be simulated is shown in Fig.3. The number of arm is m, and n_k is the number of joint of the k-th arm. In the present paper, it is assumed that the contact points between the end-effectors and the object are constant, i.e., there is no slip motion between the end-effectors and the object.

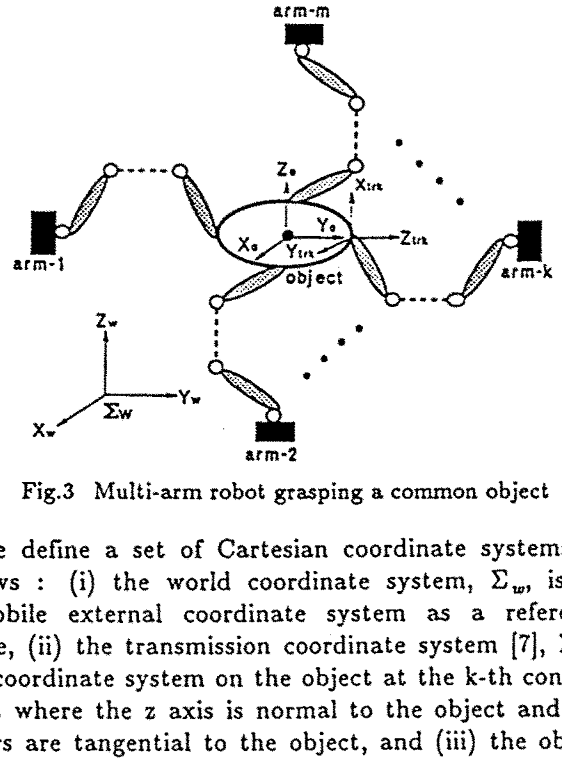

Fig.3 Multi-arm robot grasping a common object

We define a set of Cartesian coordinate systems as follows : (i) the world coordinate system, Σ_w, is an immobile external coordinate system as a reference frame, (ii) the transmission coordinate system [7], Σ_{trk}, is a coordinate system on the object at the k-th contact point where the z axis is normal to the object and the others are tangential to the object, and (iii) the object coordinate system, Σ_o, is a mobile coordinate system according to the motion of the object.

A. Kinematic Relationships of Multi-Arm Robots

The kinematic relationships of the multi-arm robot grasping a common object can be summarized in Fig.4.

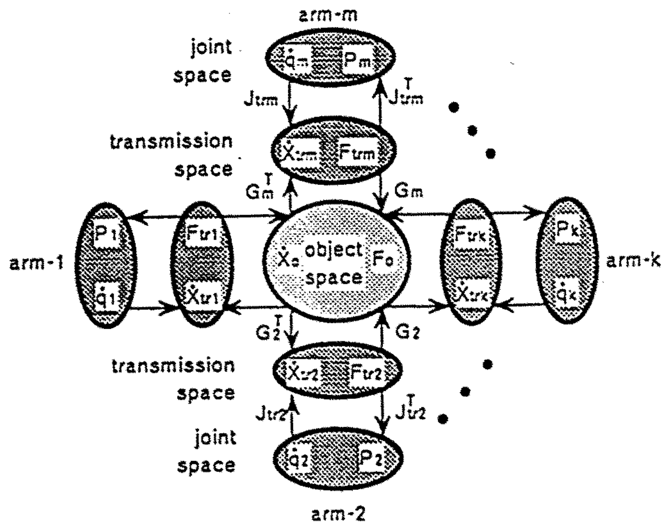

Fig.4 Kinematics of the multi-arm robot

The matrix $G_k \in R^{l \times l_{ck}}$ specifies the relationships between the object force/motion and the transmission force/motion of the k-th arm, depending on the location of the contact points, the contact type and the reference point on the object such as the center of mass. l_{ck} is the number of the end-effector directions of the k-th arm constrained by the object, and l is the dimension of the task space. The matrix G_k is given by

$$G_k = B_k R^o_{trk} H_k^T \,, \tag{30}$$

where R^o_{trk} is the rotation matrix from the transmission coordinate system, Σ_{trk}, to the object coordinate system, Σ_o. Furthermore, the matrix B_k is given by

$$B_k = \begin{bmatrix} E & 0 \\ (r_{ck})\,\chi & E \end{bmatrix} \,, \tag{31}$$

where $E \in R^{3\times3}$ denotes a unit matrix, $0 \in R^{3\times3}$ is the null matrix, $r_{ck} = [r_{ckx} \; r_{cky} \; r_{ckz}]^T$ is the position vector of the k-th contact point from the origin of the object coordinate system, Σ_o, and $(r_{ck})\,\chi \in R^{3\times3}$ is an anti-symmetric matrix obtained by applying the cross operator, " χ ", to the position vector, r_{ck}, as the following

$$(r_{ck})\,\chi = \begin{bmatrix} 0 & -r_{ckz} & r_{cky} \\ r_{ckz} & 0 & -r_{ckx} \\ -r_{cky} & r_{ckx} & 0 \end{bmatrix} . \tag{32}$$

The matrix $H_k \in R^{l_{ck} \times l}$ expresses the filter characteristics which filter out some forces/moments of the k-th end-effector and transmit other forces/moments to the object depending on the contact type.

The matrix J_{trk} is the Jacobian matrix relating the joint displacements to the transmission displacements of the k-th arm, represented in the transmission coordinate system, Σ_{trk}, and given by

$$J_{trk} = H_k R_w^{trk} J_{wk} \;\in R^{l_{ck} \times n_k} \,, \tag{33}$$

where $J_{wk} \in R^{l \times n_k}$ is the Jacobian matrix relating the joint displacements to the end-effector displacements of the k-th arm, represented in the world coordinate system, Σ_w, and R_w^{trk} is the rotation matrix from the world coordinate system, Σ_w, to the transmission coordinate system, Σ_{trk}.

B. Motion Equations

The dynamic equations of the multi-arm robot grasping a common object are formed by the dynamic equation of each arm, the motion equation of the object and the constraint equation relating the object motion and the motion of each arm.

When the multi-arm robot grasping a common object, multiple closed-chain mechanism will be formed. Each arm and the object can be considered as an open-chain with kinematic constraints on its end-effector. Therefore, from (27), the dynamic equation of the k-th arm is given by

$$\begin{bmatrix} W_k & -J_{trk}^T \\ -J_{trk} & 0 \end{bmatrix} \begin{bmatrix} \ddot{q}_k \\ \lambda_{trk} \end{bmatrix} = \begin{bmatrix} P_k + Y_k - V_k^T \\ \dot{J}_{trk}\dot{q}_k - \ddot{X}_{trk} \end{bmatrix} , \tag{34}$$

where $q_k = [q_{1k} \; q_{2k} \; \cdots \; q_{n_k k}]^T$ is the generalized coordinate vector of the k-th arm, $\ddot{X}_{trk}$ and λ_{trk} are the vector of the transmission acceleration of the k-th arm and the vector of the forces/moments exerted on the constrained end-effector of the k-th arm, respectively, which are represented in the transmission coordinate system, Σ_{trk}, and $\dot{J}_{trk}\dot{q}_k$ is computed by

$$\dot{J}_{trk}\dot{q}_k = H_k(\dot{R}_w^{trk} J_{wk} + R_w^{trk}\dot{J}_{wk}\,)\;\dot{q}_k \,. \tag{35}$$

The motion equation of the object is given by

$$\begin{bmatrix} m_o E & 0 \\ 0 & I_o \end{bmatrix} \ddot{X}_o = \begin{bmatrix} m_o g \\ -\omega_o \times (I_o \cdot \omega_o) \end{bmatrix} + F_o \,, \tag{36}$$

where m_o, I_o and ω_o are the mass, the moment inertia tensor and the angular velocity vector of the object, respectively. $\ddot{X}_o$ and F_o are the vector of the object accelerations and the vector of the forces/moments acting on the object, respectively, which are represented in the object coordinate system, Σ_o.

Since the object and the end-effectors form the parallel link structure as shown in Fig.4, the net forces/moments acting on the object is given by

$$F_o = \sum_{k=1}^{m} G_k F_{trk} \,. \tag{37}$$

On the other hand, the relationships between the transmission velocity of the k-th arm and the object velocity is given by

$$\dot{X}_{trk} = G_k^T \dot{X}_o . \quad (38)$$

Because of the matrix G_k^T is a constant matrix, then we can obtain

$$\ddot{X}_{trk} = G_k^T \ddot{X}_o . \quad (39)$$

Equations (34), (36), (37) and (39) express the dynamic equations of the multi-arms robot grasping a common object, where (39) can be interpreted as a constraint equation of each arm.

Consequently, the algorithm for the forward dynamics of the multi-arm robot proposed in this paper is summarized as the following:

step 1 At the time, t, the generalized coordinate vector of positions, $q_k(t)$, velocities, $\dot{q}_k(t)$, and the vector of generalized forces, $P_k(t)$, of each arm are given.

step 2 Using (36), compute the object acceleration, $\ddot{X}_o(t)$, where $F_o(t)$ is computed using (37). $F_{trk}(t)$ in (37) is computed by

$$F_{trk}(t) = (J_{trk}^T(t))^+ P_k(t) , \quad (40)$$

where "+" denotes the pseudo-inverse matrix. Then, using (39), compute the transmission acceleration of each arm, $\ddot{X}_{trk}(t)$.

step 3 Using (34), compute $\ddot{q}_k(t)$ and $\lambda_{trk}(t)$ for each arm, and then integrate the $\ddot{q}_k(t)$ to obtain $\dot{q}_k(t+\Delta t)$ and $q_k(t+\Delta t)$. Increase t by Δt, then return to step (1). Note that $P(t+\Delta t)$ is given according to the control law.

On the other hand, the inverse dynamics of the multi-arm robot proposed in this paper is summarized as the following:

step 1 The desired object trajectory, $X_o^d(t)$, and the desired forces/moments exerted on the constrained end-effector of the k-th arm, $\lambda_{trk}^d(t)$, are given. The object acceleration, $\ddot{X}_o(t)$, is derived from the given object trajectory.

step 2 The transmission acceleration of the k-th arm, $\ddot{X}_{trk}(t)$, is computed using (39). Then, the joint acceleration of each arm $\ddot{q}_k(t)$ is computed by

$$\ddot{q}_k(t) = (J_{trk}(t))^+ [\ddot{X}_{trk}(t) - \dot{J}_{trk}(t)\dot{q}_k(t)] . \quad (41)$$

step 3 Compute the joint torque of each arm, $P_k(t)$, using the following equation

$$P_k(t) = W_k(t)\ddot{q}_k(t) - Y_k(t) + V_k^T(t) - J_{trk}^T(t)\lambda_{trk}(t) . \quad (42)$$

Increase t by Δt, then return to step (1).

As we can see, the computation of joint acceleration, $\ddot{q}_k$, of each arm can be performed independently. It means that the dynamics of each arm can be simulated in a parallel way. On the other hand, by introducing the transmission space in the kinematic relationships of the multi-arm robot enable us to express the contact type between the end-effectors of each arm and the object using the matrix H_k in (37), such that the simulation method presented in this paper can be applied for various contact types between the end-effectors and the object.

V. Numerical Analysis

To demonstrate the effectiveness of the proposed method for various contact types, a numerical examples was performed for the case of the cooperative motion of a planar dual-arm robot ($n_1=n_2=3$; $l=3$) grasping a common object.

The contact type between the arm-1 and the object is the rigid grasping, i.e., all of the end-effector forces/moments can be transmitted to the object ($l_{c1} = 3$) and between the arm-2 and the object is the point contact with friction type, i.e., the end-effector forces can be transmitted in any direction to the object, but the end-effector moment cannot be transmitted ($l_{c1} = 2$). The link parameters of the dual-arm robot and the object parameters are shown in Table I.

TABLE I
Link Parameters of A Planar Dual–Arm Robot and the Object Parameters

	arm-k (k=1,2) link i (i=1,···,6)	object
length (m)	0.2000	0.1800
mass (kg)	0.5000	5.0000
center of mass (m)	0.1000	0.0900
moment of inertia (kg·m²)	0.0015	0.5000

The initial posture is shown in Fig.5, where $X_o(0) = [0.203225\ (m)\ \ 0.598174\ (m)\ \ 0.0\ (rad)]^T$. The task objective is to move the object from its initial position to the desired final position $X_o^d(t_f) = [0.203225\ (m)\ \ 0.498174\ (m)\ \ \pi/3\ (rad)]^T$ and $t_f = 1.0$ sec.

We choose a PD controller in the task space for positioning the object, where the joint torques of each arm are calculated using the following equations

$$P_k(t) = J_{trk}^T F_{trk}(t) , \quad (43)$$

$$F_{tr}(t) = G^+ [K(X_o^d(t) - X_o(t)) + B(\dot{X}_o^d(t) - \dot{X}_o)] , \quad (44)$$

where $F_{tr}(t) = [F_{tr1}^T(t)\ \ F_{tr2}^T(t)]^T$, $G = [G_1\ \ G_2]$ and G_k (k=1,2) is given in (30). $K \in R^{3\times3}$ and $B \in R^{3\times3}$ are the position and the velocity feedback gain matrices, given as $K = diag.$ [100.0 (N/m) 100.0 (N/m) 100.0 ($N.m/rad$)] and $B = diag.$ [10.0 ($N.sec/m$) 10.0 ($N.sec/m$) 10.0 ($N.m.sec/rad$)], where "$diag.$ [ ]" denotes a diagonal matrix.

The desired trajectory of the object is given by

$$X_{o1}^{d}(t) = 0.203225 \quad (m), \tag{45}$$

$$X_{o2}^{d}(t) = 0.598174 - 1.0t^3 + 1.5t^4 - 0.6t^5 \quad (m), \tag{46}$$

$$X_{o3}^{d}(t) = 10.0\pi/3t^3 - 5.0\pi t^4 + 2.0\pi t^5 \quad (rad). \tag{47}$$

The sampling time Δt is set to 0.0001 sec.

Fig.6 shows the stick pictures of the dual-arm robot under the PD controller, and Table II shows the angle between the axes of the last link of each arm and the axes of the object (the contact-angle).

It can be seen that for the rigid grasping, the contact-angle between the end-effector of arm-1 and the object is constant. On the other hand, for the point contact with friction, since the end-effector's moment of arm-2 cannot be transmitted to the object, the end-effector can rotate freely. As a result, the contact-angle between the end-effector of arm-2 and the object changes during control.

VI. Conclusion

We have proposed the dynamic simulation method of a multi-arm robot using Appel's method. It was shown that the dynamic equation for a multi-arm robot grasping a common object is formed by the dynamic equation of each arm, the motion equation of the object and the constraint equation.

The dynamic equation of each arm has a form identical to the dynamic equation of an open-loop single-arm with kinematic constraints on its end-effector, where the end-effector's constraint equation is obtained from the object motion. By using the Appel's method, each arm of the multi-arm robot can be simulated in a parallel way, such that the parallel computation can be implemented. Also, The proposed method can be applied for various contact types between the end-effectors and the object, since the mechanism of the forces/moments transmission between the end-effectors and the object is taken into consideration.

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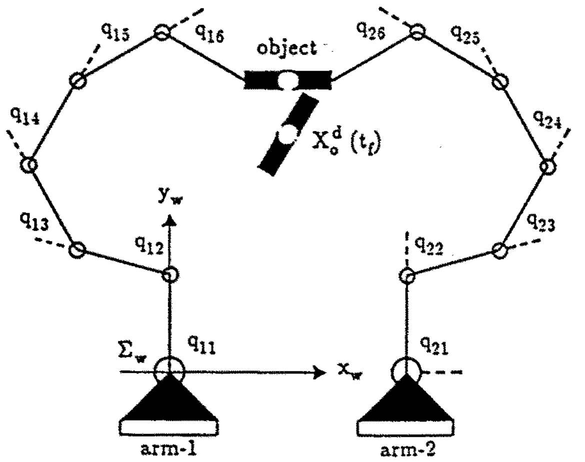

$X_o(0) = [0.203225\ (m)\ \ 0.598174\ (m)\ \ 0.0\ (rad)\ ]^T$

$X_o^d(t_f) = [0.203225\ (m)\ \ 0.498174\ (m)\ \ \pi/3\ (rad)\ ]^T$

Fig.5 Planar dual-arm robot grasping a common object: initial posture

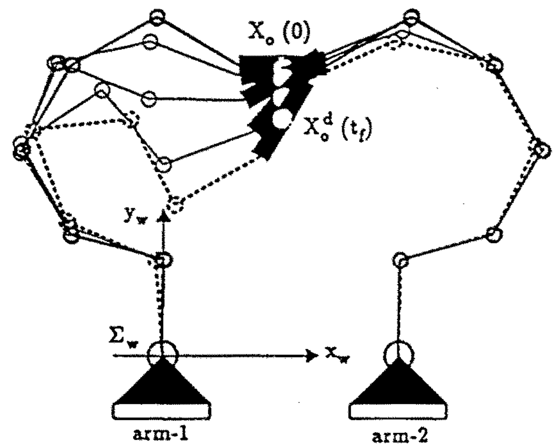

Fig.6 Stick pictures of a planar dual-arm robot during the object movement

TABLE II
ANGLES BETWEEN THE AXES OF THE LAST LINK OF EACH ARM AND THE AXES OF THE OBJECT

time (s)	Arm-1	Arm-2
0.0	0.523599 (rad)	-0.523599 (rad)
0.2	0.523599 (rad)	-0.506423 (rad)
0.4	0.523599 (rad)	-0.309309 (rad)
0.6	0.523599 (rad)	0.119070 (rad)
0.8	0.523599 (rad)	0.510500 (rad)
1.0	0.523599 (rad)	0.613626 (rad)