

# MOTOR IMPEDANCE AND INVERSE KINEMATICS IN MUSCULOSKELETAL SYSTEMS

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**ABSTRACT** - A method for resolving kinematic redundancies of musculoskeletal systems is discussed. First, the relationships between muscle force and end-point motion are derived by defining two kind of Jacobian matrices. Then it is shown that the motor impedance allows to transform not only the end-point displacement but also the end-point velocity and acceleration into the joint and muscle coordinates.

## INTRODUCTION

The CNS plans arm movements by specifying a trajectory of the hand in Euclidean(end-point) space[1]. Consequently, it is necessary to derive the coordinated patterns of joint and muscle levels, which realize the end-point trajectory. This is called the inverse kinematics problem and is also one of the most important problems in robot control[2].

In general, since the number of degrees of freedom in the human arm is larger than one in the task space, there exists an infinite number of solutions to the inverse kinematics problem. In order to determine a definite solution, it is necessary to impose some constraint on the transformation.

Mussa Ivaldi showed that the joint stiffness could act as a constraint condition to resolve the inverse kinematics problem of redundant arm[3]. In this paper, we emphasize the role of motor impedance which transmits the muscle force to the end-point motion. First, the relationships between muscle force and end-point motion are derived. Then it is shown that the motor impedance allows to transform not only the end-point displacement but also the end-point velocity and acceleration into the joint and muscle coordinates.

## MULTI-JOINT ARM MOVEMENTS

We consider a multi-joint arm having  $n$  joints. Let the position vectors in the joint coordinates and the end-point coordinates be denoted as  $\theta \in R^n$  and  $X \in R^r$  respectively. The transformation from  $\theta$  to  $X$  is given by nonlinear equation,

$$X = p(\theta) . \quad (1)$$

The Jacobian matrix  $J$  is the locally linearized transformation matrix which is defined by[2]

$$dX = J(\theta)d\theta . \quad (2)$$

The principle of duality between velocity and force in the mechanics leads to the equation,

$$\tau = J^T F \quad (3)$$

where  $\tau \in R^n$  and  $F \in R^r$  are the force vectors in the joint coordinates and the end-point coordinates, respectively.

On the other hand, arm movements are generated by  $m$  muscles which act on the joints. Let the muscle length vector and the muscle force vector be denoted as  $L \in R^m$  (define that its extending direction is positive) and  $f \in R^m$  (define that the contracting direction is positive). The muscle length vector  $L$  is given by nonlinear function of joint angle vector  $\theta$ ,

$$L = q(\theta) . \quad (4)$$

Locally linearizing (4) around a posture  $\theta$ , we can see,

$$dL = G(\theta)d\theta . \quad (5)$$

The transformation  $G$  is another Jacobian matrix which determines the relationships between joint and muscle movements. Similarly to (3), the transformation from  $f$  to  $\tau$  is given by

$$\tau = -G^T f . \quad (6)$$

Consequently, the relationships among muscle, joint and end-point movements can be represented by two kind of Jacobian matrices.

## MOTOR IMPEDANCE AND MUSCLE FORCE

The motor impedance which is a general term for stiffness, viscosity and inertia provides the static and dynamic relations between force and motion[4].

First, consider the stiffness relationships among muscle, joint and end-point level. The three kind of stiffness matrices are defined as follows,

$$1) \text{ end-point level ; } F = -K_e dX \quad (7)$$

$$2) \text{ joint level ; } \tau = -K_j d\theta \quad (8)$$

$$3) \text{ muscle level ; } f = K_m dL \quad (9)$$

where  $dX = X - X^e$ ,  $d\theta = \theta - \theta^e$  and  $dL = L - L^e$ .  $X^e$ ,  $\theta^e$  and  $L^e$  are equilibrium points of the corresponding vectors. The muscle stiffness matrix  $K_m$  is adjustable by the viscoelastic properties of muscles and the proprioceptive reflexes[5].

The stiffness relationships between each level are derived using (2)-(5),

$$K_j = J^T K_e J \quad (10)$$

$$= G^T K_m G . \quad (11)$$

Then, the transformations of the compliance

matrices which are the inverse of the corresponding stiffness matrices are given by

$$C_e = JC_j J^T \quad (12)$$

$$C_m = GC_j G^T \quad (13)$$

Fig.1(a) shows the transformations between the force and motion by the stiffness and compliance matrices. From the figure, we find the pathway to be followed in order to go from the muscle force to the end-point displacement. The corresponding relation is written as

$$dX = JC_j G^T f, \quad (14)$$

which gives the transformation of the muscle force to the end-point displacement.

Next, consider the transformation of the inertia matrix. In general, the motion equation of the multi-joint arm can be written as

$$M(\theta) \ddot{\theta} + h(\theta, \dot{\theta}) + g(\theta) = \tau, \quad (15)$$

where,  $M(\theta)$  is non-singular inertia matrix,  $h(\theta, \dot{\theta})$  is Coriolis and centrifugal term and  $g(\theta)$  is the gravity term. Now, assume that the arm is at rest and the gravity term doesn't exist, such as planar movements. The transformation from the joint acceleration to the end-point one is given by

$$\ddot{X} = J \ddot{\theta}. \quad (16)$$

Then we have

$$\ddot{X} = -JM^{-1}G^T \tau, \quad (17)$$

which defines the transformation of the muscle force to the end-point acceleration. Substituting (3) into (17) yields

$$\ddot{X} = JM^{-1}J^T f \quad (18)$$

where the matrix  $JM^{-1}J^T$  is called the mobility[4]. Fig.1(b) shows the transformation between the force and acceleration by the inertia matrices. In the case that there exists the gravity force, we can replace the joint torque  $\tau$  to  $\tilde{\tau} = \tau - g(\theta)$  which means compensation of the gravity force by the muscle force.

#### INVERSE KINEMATICS WITH IMPEDANCE CONSTRAINTS

Transforming the joint movements into the muscle movements can be performed using the Jacobian matrix  $G$ . However, it is not easy to transform the

end-point movement to the joint movement due to the arm redundancy.

In Fig.1(a), there exists a pathway that starts from the end-point displacement  $dX$  and reaches to the joint displacement  $d\theta$  through the end-point force  $F$  and the joint torque  $\tau$ .

$$d\theta = C_j J^T K_e dX \quad (18)$$

Assuming that the matrices  $K_e$  and  $K_j$  are non-singular, we have[3]

$$d\theta = K_j^{-1} J^T (JK_j^{-1} J^T)^{-1} dX. \quad (19)$$

In a similar way, the transformation from the end-point acceleration to the joint angle acceleration is given by

$$\ddot{\theta} = M^{-1} J^T (JM^{-1} J^T)^{-1} \ddot{X}. \quad (20)$$

(19) and (20) allow to resolve the inverse kinematics problem of the redundant arm. On the inverse kinematics problem of the redundant arm, Whitney proposed to evaluate the solution to (2) by minimizing the quadratic cost function,

$$Q(d\theta) = d\theta^T W d\theta \quad (21)$$

where  $W \in R^{m \times m}$  is a symmetric positive definite weighting matrix[6]. This yields the instantaneous inverse kinematics of motion

$$d\theta = W^{-1} J^T (JW^{-1} J^T)^{-1} dX. \quad (22)$$

Comparing (22) to (19) and (20), it is known that the weighting matrix  $W$  is replaced with the joint stiffness matrix  $K_j$  and the inertia matrix  $M$ , respectively. This means that we can gain the instantaneous inverse kinematics weighted by the mechanical properties of the arm, i.e., by the motor impedance.

#### CONCLUSION

In this paper, first, the relationships among muscle, joint and end-point movements were derived by two kind of the Jacobian matrices. Then, it was shown that the motor impedance played the role as the constraints to solve the inverse kinematics problem of the redundant arm. Future research will be directed how to plan the end-point impedance appropriate to the specific tasks. This is one of the common problems to realize the impedance regulation in human arm and robotic manipulator.

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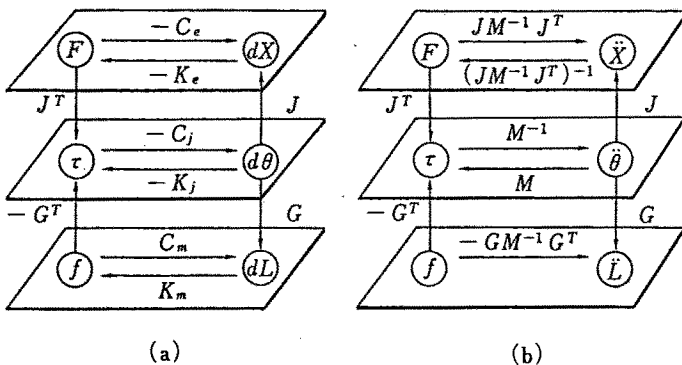


Fig.1 The Impedance Relationship among Muscle, Joint and End-Point Level